Sections: 9.4
Due: 5 June 2018

1. 5 Verify

$$
\left\{\left[\begin{array}{c}
-e^{t} \\
e^{t} \\
2 e^{t}
\end{array}\right],\left[\begin{array}{c}
-2 e^{2 t} \\
e^{2 t} \\
4 e^{2 t}
\end{array}\right],\left[\begin{array}{c}
-e^{3 t} \\
e^{3 t} \\
4 e^{3 t}
\end{array}\right]\right\}
$$

is a fundamental solution set to

$$
\mathbf{x}^{\prime}(t)=\left[\begin{array}{ccc}
1 & 2 & -1 \\
1 & 0 & 1 \\
4 & -4 & 5
\end{array}\right] \mathbf{x}(t)
$$

Recall a fundamental solution set for a system of $n$ first order equations is a set of $n$ linearly independent solutions. You should show each vector function is a solution. You should also show that they are linearly independent.
2. Consider the equation $\mathbf{x}^{\prime}(t)=\mathbf{A} \mathbf{x}(t)$ with $\mathbf{A}=\left[\begin{array}{cc}-1 & 2 \\ -1 & -3\end{array}\right]$.
(a) 3 Show

$$
\left\{\left[\begin{array}{c}
2 e^{-2 t} \cos t \\
-e^{-2 t}(\cos t+\sin t)
\end{array}\right],\left[\begin{array}{c}
2 e^{-2 t} \sin t \\
e^{-2 t}(\cos t-\sin t)
\end{array}\right],\right\}
$$

is a fundamental solution set for $\mathbf{x}^{\prime}(t)=\mathbf{A x}(t)$.
(b) 1 Find a fundamental matrix for $\mathbf{x}^{\prime}(t)=\mathbf{A x}(t)$.
(c) 1 Solve the initial value problem $\mathbf{x}^{\prime}(t)=\mathbf{A} \mathbf{x}(t), \mathbf{x}(0)=\left[\begin{array}{l}1 \\ 4\end{array}\right]$.

