**Math 274 Homework** **Name:** ____________________

Sections: 9.5,9.6

Due: 6 June 2018

Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Find a fundamental matrix for

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{x}.$$

$$\begin{vmatrix} 1-r & 1 \\ 1 & 1-r \end{vmatrix} = (1-r)^2 - 1$$

$$= r^2 - 2r$$

$$= r(r-2)$$

$r = 0$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \vec{0}$$

$r = 2$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \vec{0}$$

$$\underline{X} = \begin{bmatrix} 1 & e^{2t} \\ -1 & e^{2t} \end{bmatrix}$$

2. $\boxed{3}$ Find the solution to the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$\begin{vmatrix} 1-r & 2 \\ 0 & 3-r \end{vmatrix} = (1-r)(3-r)$$

$r = 1$

$$\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \vec{0}$$

$r = 3$

$$\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \vec{0}$$

$$\underline{X} = \begin{bmatrix} e^{t} & e^{3t} \\ 0 & e^{3t} \end{bmatrix}$$

$$\underline{X}(0) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\underline{X}^{-1}(0) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\underline{X}^{-1}(0)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\vec{X} = \underline{X}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{t} - e^{3t} \\ -e^{3t} \end{bmatrix}$$

3. $\boxed{2}$ Find a fundamental matrix for

$$\mathbf{x}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x}.$$

$$\begin{vmatrix} -r & -2 \\ 2 & -r \end{vmatrix} = r^2 + 4 = 0$$

$r = \pm 2i$

$$\Psi = \begin{bmatrix} \cos 2t & \sin 2t \\ \sin 2t & -\cos 2t \end{bmatrix}$$

$r = 2i$

$$\begin{bmatrix} -2i & -2 \\ 2 & -2i \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} = \vec{0}$$

– OR –

$$\Psi = \begin{bmatrix} -\sin 2t & \cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$$

$\begin{bmatrix} i \\ 1 \end{bmatrix}$ is also a solution / eigenvector

4. $\boxed{3}$ Find the solution to the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} -3 \\ 5 \end{bmatrix}.$$

$$\begin{vmatrix} -r & 1 \\ -2 & -2-r \end{vmatrix} = r^2 + 2r + 2$$

$$= (r+1)^2 + 1 = 0$$

$r = -1 \pm i$

$r = -1 + i$

$$\begin{bmatrix} 1-i & 1 \\ -2 & -1-i \end{bmatrix} \begin{bmatrix} 1 \\ -1+i \end{bmatrix}$$

or

$$\begin{bmatrix} 1+i \\ -2 \end{bmatrix}$$

$$\vec{x} = C_1 e^{-t}\left( \cos t \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$+ C_2 e^{-t}\left( \sin t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \cos t \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$\vec{x}(0) = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

so $C_1 = -3$

$C_2 = 2$

$$\vec{x} = -3e^{-t} \begin{bmatrix} \cos t \\ -\cos t - \sin t \end{bmatrix} + 2e^{-t} \begin{bmatrix} \sin t \\ -\sin t + \cos t \end{bmatrix}$$

$$= e^{-t} \begin{bmatrix} 2\sin t - 3\cos t \\ 5\cos t + \sin t \end{bmatrix}$$