Math 274 Homework Name: ____________

Sections: 2.2,2.3

Due: 16 May 2018

Point values in $\boxed{\text{boxes}}$.

1. Consider the differential equation

$$\frac{dy}{dx} = \frac{x(y^2-1)}{y^2}. \qquad (1)$$

(a) $\boxed{3}$ Find an implicit general solution to (1).

Lost $y \equiv 1$, $y \equiv -1$ $\longrightarrow$ $\displaystyle\int \frac{y^2}{y^2-1}\,dy = \int x\,dx$

Note: $\displaystyle\frac{y^2}{y^2-1} = \frac{y^2-1}{y^2-1} + \frac{1}{y^2-1} = 1 + \frac{1}{2}(y-1) - \frac{1}{2}(y+1)$

$$\int \left(1+\tfrac{1}{2}(y-1)-\tfrac{1}{2}(y+1)\right)dy = \frac{x^2}{2}+C$$

$$\underbrace{y + \tfrac{1}{2}\ln|y-1| - \tfrac{1}{2}\ln|y+1| = \frac{x^2}{2}+C}_{(*)} \quad \text{OR} \quad y \equiv 1 \quad \text{OR} \quad y \equiv -1$$

(b) $\boxed{1}$ Find a solution[1] to (1) satisfying $y(0) = 1$.

$y \equiv 1$. Note: there is no C so that $(*)$ satisfies $y(0)=1$

2. $\boxed{3}$ Find an explicit general solution to

$$xy' - 2y = x^3.$$

$y' - \frac{2}{x}y = x^2$ so $\mu(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln|x|} = x^{-2}$

$$x^{-2}y' - 2x^{-3}y = 1$$

$$(x^{-2}y)' = 1$$

$$x^{-2}y = \int dx = x + C$$

$$y = x^2(x+C)$$

[1] Did you lose anything in part (a)?

3. $\boxed{3}$ Consider the equation

$$\frac{dy}{dx} = \frac{y-x}{x+y}. \tag{2}$$

Equation (2) is neither separable nor linear. Please read the first few pages of Section 2.6 regarding Homogeneous equations. The equation is homogeneous since it can be rewritten as

$$\frac{dy}{dx} = \frac{y/x - 1}{1 + y/x}.$$

Using the substitution $v = y/x$ so that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ converts (2) into

$$v + x\frac{dv}{dx} = \frac{v-1}{1+v}. \tag{3}$$

Equation (3) is now separable. Find an implicit general solution in terms of v and x.

$$x\frac{dv}{dx} = \frac{v-1}{1+v} - v = \frac{v-1-v-v^2}{1+v} = -\frac{v^2+1}{v+1}$$

$$\int \frac{v+1}{v^2+1}\,dv = \int -\frac{dx}{x}$$

$$\int\left(\frac{v}{v^2+1} + \frac{1}{v^2+1}\right)dv = -\ln|x| + C$$

$$\frac{1}{2}\ln(v^2+1) + \arctan v = C - \ln|x|$$

Using the original substitution $v = y/x$, find an implicit general solution to (2) in terms of the original variables, y and x.

$$\frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2 + 1\right) + \arctan\left(\frac{y}{x}\right) = C - \ln|x|$$