Due: 16 May 2018

1. Consider the differential equation

$$
\begin{equation*}
\frac{d y}{d x}=\frac{x\left(y^{2}-1\right)}{y^{2}} . \tag{1}
\end{equation*}
$$

(a) 3 Find an implicit general solution to (1).
(b) 1 Find a solution ${ }^{1}$ to (1) satisfying $y(0)=1$.
2. 3 Find an explicit general solution to

$$
x y^{\prime}-2 y=x^{3} .
$$

[^0]3. 3 Consider the equation
\[

$$
\begin{equation*}
\frac{d y}{d x}=\frac{y-x}{x+y} . \tag{2}
\end{equation*}
$$

\]

Equation (2) is neither separable nor linear. Please read the first few pages of Section 2.6 regarding Homogeneous equations. The equation is homogeneous since it can be rewritten as

$$
\frac{d y}{d x}=\frac{y / x-1}{1+y / x}
$$

Using the substitution $v=y / x$ so that $\frac{d y}{d x}=v+x \frac{d v}{d x}$ converts (2) into

$$
\begin{equation*}
v+x \frac{d v}{d x}=\frac{v-1}{1+v} . \tag{3}
\end{equation*}
$$

Equation (3) is now separable. Find an implicit general solution in terms of $v$ and $x$.

Using the original substitution $v=y / x$, find an implicit general solution to (2) in terms of the original variables, $y$ and $x$.


[^0]:    ${ }^{1}$ Did you lose anything in part (a)?

