Math 274 Homework **Name:** ______________

Sections: 2.6,3.2,Complex

Due: 17 May 2018

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Evaluate $\operatorname{Re}\left((1-i)e^{i\theta}\right)$.

$$(1-i)e^{i\theta} = (1-i)(\cos\theta + i\sin\theta) = (\cos\theta + \sin\theta) + i(\sin\theta - \cos\theta)$$

$$\text{so } \operatorname{Re}\left((1-i)e^{i\theta}\right) = \cos\theta + \sin\theta$$

2. $\boxed{2}$ A balloon is initially filled with 1 L of pure helium. A tank with a mix of 25% helium and 75% oxygen is being used to fill the balloon at a rate of 0.4 L/min. The seal between the tank and the balloon is not airtight and the resulting mixture inside the balloon (assume the gas inside the balloon is well mixed by the magic of mathematics) is escaping at a rate of 0.3 L/min. The balloon pops when it reaches a volume of 5 L. For $t \in [0, 40)$ let the amount of helium, in L, in the the balloon be $h(t)$. Write an initial value problem, i.e. a differential equation with initial data, that models the amount of helium in the balloon. **Do Not Solve.**

$$\frac{dh}{dt} = (0.4)\left(\frac{1}{4}\right) - (0.3)\left(\frac{h}{1+0.1t}\right), \quad h(0) = 1$$

3. Find general solutions for the following first order equations.

(a) $\boxed{2}$

$$y' = \frac{x^2 + 3xy + y^2}{x^2}$$

$$y' = 1 + 3\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$$

Let $v = \frac{y}{x}$

so $xv = y$

$$v + xv' = y'$$

Substituting gives

$$v + xv' = 1 + 3v + v^2$$

so $x\frac{dv}{dx} = 1 + 2v + v^2 = (1+v)^2$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x} \quad \text{OR } v = -1$$

$$\frac{-1}{(1+v)} = \ln|x| + C$$

$$\frac{-1}{(1+y/x)} = \ln|x| + C \quad \boxed{\text{OR } y = -x}$$

(b) $\boxed{2}$

$$y' = x\tan y$$

$$\int \frac{dy}{\tan y} = \int x \, dx \qquad \text{OR} \qquad \tan y = 0, \text{ i.e. } y \equiv k\pi, \ k \in \mathbb{Z}$$

$$\int \frac{\cos y}{\sin y}\, dy = \frac{x^2}{2} + C$$

$$\ln|\sin y| = \frac{x^2}{2} + C \qquad \boxed{\text{OR} \quad y \equiv k\pi, \ k \in \mathbb{Z}}$$

OR

$$\sin y = Ce^{x^2/2}$$ (note the lost solutions were reclaimed)

(c) $\boxed{2}$

$$y' + 2xy = 2xy^4$$

$$y^{-4}y' + 2xy^{-3} = 2x$$

$$-3y^{-4}y' - 6xy^{-3} = -6x \qquad \text{Let } v = y^{-3} \text{ so } v' = -3y^{-4}y'$$

$$v' - 6xv = -6x \qquad \mu(x) = e^{-3x^2}$$

$$\left(ve^{-3x^2}\right)' = -6xe^{-3x^2}$$

$$ve^{-3x^2} = \int -6xe^{-3x^2}\, dx = e^{-3x^2} + C$$

$$v = 1 + Ce^{3x^2} \quad \text{so} \quad y^{-3} = 1 + Ce^{3x^2} \quad \boxed{\text{or} \quad y \equiv 0}$$

(d) $\boxed{1}$ In the process of solving the three previous equations you likely lost some solutions, likely an infinite amount of them. Be sure to find them and include them in your solutions.