Math 274 Homework **Name:** ____________

Sections: 4.6,4.7 Point Values in boxes.

Due: 23 May 2018

1. [1] The Bessel equation of order one-half

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0, \quad t > 0$$

has solutions $y_1 = t^{-1/2}\cos t$ and $y_2 = t^{-1/2}\sin t$. Use the Wronskian to verify that y_1 and y_2 are linearly independent for $t > 0$.

$$W[y_1, y_2] = \begin{vmatrix} t^{-1/2}\cos t & t^{-1/2}\sin t \\ -\frac{1}{2}t^{-3/2}\cos t - t^{-1/2}\sin t & -\frac{1}{2}t^{-3/2}\sin t + t^{-1/2}\cos t \end{vmatrix}$$

$$= -\frac{1}{2}t^{-2}\cos t\sin t + t^{-1}\cos^2 t + \frac{1}{2}t^{-2}\cos t\sin t + t^{-1}\sin^2 t$$

$$= t^{-1}\cos^2 t + t^{-1}\sin^2 t = t^{-1} \neq 0.$$

2. [4] Using the above, find a general solution to

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = t^{5/2}, \quad t > 0$$

Std form

$$y'' + \frac{1}{t}y' + \left(1 - \frac{1}{4t^2}\right)y = t^{1/2}$$

$$y_p = t^{-1/2}\cos t \int \frac{-t^{1/2}\cdot t^{-1/2}\sin t}{t^{-1}}\,dt + t^{-1/2}\sin t \int \frac{t^{1/2}\cdot t^{-1/2}\cos t}{t^{-1}}\,dt$$

$$= t^{-1/2}\cos t \int (-t\sin t)\,dt + t^{-1/2}\sin t \int (t\cos t)\,dt$$

$u = t$ $dv = -\sin t\,dt$ $du = dt$ $v = \cos t$

$u = t$ $dv = \cos t\,dt$ $du = dt$ $v = \sin t$

$$= t^{-1/2}\cos t\left[t\cos t - \sin t\right] + t^{-1/2}\sin t\left[t\sin t + \cos t\right]$$

$$= t^{-1/2}\left[t\cos^2 t - \sin t\cos t + t\sin^2 t + \sin t\cos t\right] = t^{-1/2}\left[t(\cos^2 t + \sin^2 t)\right] = t^{1/2}$$

3. [5] Find a general solution for

$$ty'' - (t+1)y' + y = t^2$$

provided that $y_1 = e^t$ solves

$$ty'' - (t+1)y' + y = 0$$

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HINT: Start by finding a second linearly independent solution to the homogeneous equation, then apply Variation of Parameters.

In standard form $y'' + \left(-1 - \frac{1}{t}\right) y' + \frac{1}{t} y = 0$ (or t)

$$y_2 = e^t \int \frac{e^{\int (1 + \frac{1}{t}) dt}}{e^{2t}} dt = e^t \int \frac{e^{t + \ln t}}{e^{2t}} dt = e^t \int t e^{-t} dt$$

$u = t \quad dv = e^{-t} dt$

$du = dt \quad v = -e^{-t}$

$$= e^t \left(-t e^{-t} - e^{-t} \right)$$

$$= -t - 1$$

$$W[y_1, y_2] = \begin{vmatrix} e^t & -t-1 \\ e^t & -1 \end{vmatrix} = -e^t + (t+1)e^t = te^t$$

$$y_p = e^t \int \frac{-t(-t-1)}{te^t} dt + (-t-1) \int \frac{te^t}{te^t} dt$$

$$= e^t \int (t+1) e^{-t} - (t+1)t$$

$u = t+1 \quad dv = e^{-t} dt$

$du = dt \quad v = -e^{-t}$

$$= e^t \left[-(t+1)e^{-t} - e^{-t} \right] - (t+1)t$$

$$= -(t-1) - 1 - t^2 - t$$

$$= -t^2 - 2t - 2$$