Math 274 Homework **Name:** ____________

Sections: 7.2,7.3 Point Values in $\boxed{\text{boxes}}$.

Due: 24 May 2018

1. $\boxed{2}$ For what values of b is the mass-spring system given by

$$2x'' + bx' + 6x = 0$$

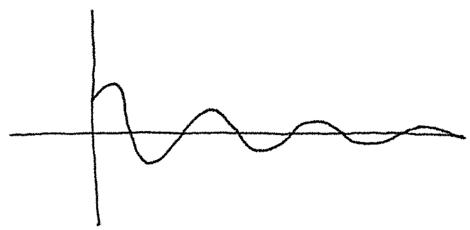
underdamped? Sketch one such solution curve satisfying $x(0) = x'(0) = 1$.

$b^2 - 4 \cdot 2 \cdot 6 < 0$

i.e. $b^2 < 36$

i.e. $b < 6$

OR $b \in [0, 6)$



2. Use the table to find the Laplace transform of the following.

(a) $\boxed{1}$ $f(t) = 4e^{-2t} + e^{2t}\sin 3t$

$$F(s) = \frac{4}{s+2} + \frac{3}{(s-2)^2 + 3^2}$$

(b) $\boxed{1}$ $g(t) = \cos(3t)\sin(5t)$ [HINT: Product to Sum identity.]

$$g(t) = \frac{1}{2}\left(\sin 8t - \sin(-2t)\right)$$

$$= \frac{1}{2}\left(\sin 8t + \sin 2t\right)$$

$$G(s) = \frac{1}{2}\left[\frac{8}{s^2+8^2} + \frac{2}{s^2+2^2}\right]$$

3. $\boxed{2}$ Use the table of Laplace transforms to show

$$\mathscr{L}\left\{\frac{1}{2}(\sin 3t - 3t\cos 3t)\right\} = \frac{27}{(s^2+9)^2}.$$

$$\mathcal{L}\left\{\frac{1}{2}\left(\sin 3t - 3t\cos 3t\right)\right\} = \frac{1}{2}\left[\frac{3}{s^2+9} + 3\frac{d}{ds}\left[\frac{s}{s^2+9}\right]\right]$$

$$= \frac{3}{2}\left[\frac{s^2+9}{(s^2+9)^2} + \frac{(s^2+9) - s(2s)}{(s^2+9)^2}\right]$$

$$= \frac{3}{2}\left[\frac{18}{(s^2+9)^2}\right]$$

$$= \frac{27}{(s^2+9)^2}$$

4. $\boxed{4}$ Apply the Laplace transform to the initial value problem

$$y'' + 3y' + y = t\cos 2t, \qquad y(0) = 1, y'(0) = -3$$

to express $Y(s) = \mathscr{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; i.e. the right-hand side should be a single combined fraction with the numerator multiplied out and the denominator factored into linear and/or irreducible quadratic terms.

Do not find the inverse Laplace transform.

$$s^2Y - s + 3 + 3(sY - 1) + Y = -\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = -\left(\frac{s^2+4-2s^2}{(s^2+4)^2}\right)$$

$$Y(s^2+3s+1) = \frac{s^2-4}{(s^2+4)^2} + \frac{s(s^2+4)^2}{(s^2+4)^2}$$

$$= \frac{s^2-4+s^5+8s^3+16s}{(s^2+4)^2}$$

$$Y = \frac{s^5+8s^3+s^2+16s-4}{(s^2+4)^2(s^2+3s+1)}$$