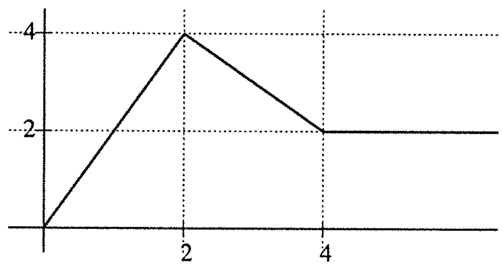
Math 274 Homework **Name:** ____________________

Sections: 7.6 Point Values in $\boxed{\text{boxes}}$.

Due: 30 May 2018

Note: Historically, section 7.6 is difficult for students. If you you have not been doing the suggested exercises, now would be a great time to start.

1. $\boxed{3}$ A graph of $y = f(x)$ is given below.



Express f as a piecewise defined function and compute its Laplace transform.

$$f(t) = \begin{cases} 2t & t < 2 \\ 6-t & 2 < t < 4 \\ 2 & t > 4 \end{cases} = 2t + u(t-2)(6-3t) + u(t-4)(t-4)$$

$$F(s) = \frac{2}{s^2} + e^{-2s}\mathscr{L}\{6-3(t+2)\} + e^{-4s}\mathscr{L}\{t+4-4\}$$

$$= \frac{2}{s^2} + e^{-2s}\left(\frac{-3}{s^2}\right) + e^{-4s}\left(\frac{1}{s^2}\right)$$

2. $\boxed{3}$ Applying the Laplace transform to the initial value problem

$$y'' + 4y = \begin{cases} 0, & t < 2 \\ 8e^t, & 2 < t \end{cases}, \qquad y(0) = 1, y'(0) = 2$$

gives

$$Y(s) = \frac{s+2}{s^2+4} + \frac{8e^{4-2s}}{(s-2)(s^2+4)}.$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$Y(s) = \frac{s}{s^2+4} + \frac{2}{s^2+4} + e^4 e^{-2s}\left[\frac{1}{s-2} - \frac{s}{s^2+4} - \frac{2}{s^2+4}\right] \leftarrow \text{see worksheet 2(b)}$$

$$y(t) = \cos 2t + \sin 2t + e^4 u(t-2)\left[e^{2(t-2)} - \cos(2t-4) - \sin(2t-4)\right]$$

3. Consider the initial value problem

$$y'' + y = \begin{cases} 3\sin 2t, & t < \pi \\ 0, & \pi < t \end{cases}, \qquad y(0) = 0, y'(0) = 0$$

(a) $\boxed{3}$ Use the method of Laplace transforms to solve the initial value problem.

$$s^2 Y + Y = \mathcal{L}\{3\sin 2t - u(t-\pi)\,3\sin 2t\}$$

$$= \frac{6}{s^2+4} - e^{-\pi s}\mathcal{L}\{3\sin(2t+2\pi)\}$$

Note: $\sin(2t+2\pi) = \sin 2t$

$$= \frac{6}{s^2+4} - e^{-\pi s}\frac{6}{s^2+4}$$

$$Y = \frac{6}{(s^2+1)(s^2+4)}\left(1 - e^{-\pi s}\right)$$

$$\frac{6}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

so $6 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$

Let $s = i$: $6 = (Ai+B)(3)$

so $A=0$, $B=2$

Eq. Coeff

s^3: $0 = A + C$ so $C = 0$

s^0: $6 = 4B + D$ so $D = -2$

so

$$Y = \left[\frac{2}{s^2+1} - \frac{2}{s^2+4}\right]\left(1 - e^{-\pi s}\right)$$

$$y = 2\sin t - \sin 2t - u(t-\pi)\left[2\sin(t-\pi) - \sin(2t-2\pi)\right]$$

$$= 2\sin t - \sin 2t - u(t-\pi)\left[-2\sin t - \sin 2t\right]$$

$\sin(t-\pi) = -\sin t$

(b) $\boxed{1}$ Express your solution as a simplified (i.e. combine like terms) piecewise defined function. You may find it useful to know $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

$$y(t) = \begin{cases} 2\sin t - \sin 2t, & 0 < t < \pi \\ 4\sin t, & t > \pi \end{cases}$$