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Due: 30 May 2018
Note: Historically, section 7.6 is difficult for students. If you you have not been doing the suggested exercises, now would be a great time to start.

1. 3 A graph of $y=f(x)$ is given below.


Express $f$ as a piecewise defined function and compute its Laplace transform.
2. 3 Applying the Laplace transform to the initial value problem

$$
y^{\prime \prime}+4 y=\left\{\begin{array}{ll}
0, & t<2 \\
8 e^{t}, & 2<t
\end{array}, \quad y(0)=1, y^{\prime}(0)=2\right.
$$

gives

$$
Y(s)=\frac{s+2}{s^{2}+4}+\frac{8 e^{4-2 s}}{(s-2)\left(s^{2}+4\right)}
$$

Determine $y(t)=\mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.
3. Consider the initial value problem

$$
y^{\prime \prime}+y=\left\{\begin{array}{ll}
3 \sin 2 t, & t<\pi \\
0, & \pi<t
\end{array}, \quad y(0)=0, y^{\prime}(0)=0\right.
$$

(a) 3 Use the method of Laplace transforms to solve the initial value problem.
(b) 1 Express your solution as a simplified (i.e. combine like terms) piecewise defined function. You may find it useful to know $\sin (\alpha-\beta)=\sin \alpha \cos \beta-\cos \alpha \sin \beta$.

