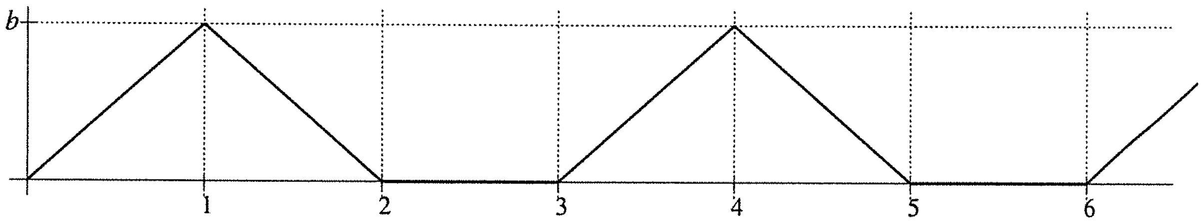
Math 274 Homework
Sections: 7.7,7,8,7,9
Due: 31 May 2018

Name: ______________________

Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ For $b > 0$, a graph of $y = f(t)$ is given below.



Compute the Laplace transform of $f(t)$.

$$f_3 = bt + u(t-1)\left[(2b - bt) - bt\right] + u(t-2)\left[bt - 2b\right]$$

$$F_3 = \frac{b}{s^2} + e^{-s}\left(\frac{-2b}{s^2}\right) + e^{-2s}\left(\frac{b}{s^2}\right)$$

$$F = \frac{b\left(1 - 2e^{-s} + e^{-2s}\right)}{s^2\left(1 - e^{-3s}\right)}$$

2. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y' - 2y = g(t), \qquad y(0) = 3.$$

(a) $\boxed{1}$ Find the solution. Express your solution in terms of a convolution.

$$sY - 3 - 2Y = G$$

$$Y(s-2) = G + 3$$

$$Y = \frac{G}{s-2} + \frac{3}{s-2}$$

so $y = g(t) * e^{2t} + 3e^{2t}$

(b) $\boxed{2}$ If $g(t) = 2t + 3$, find the solution by evaluating the convolution integral you found in (a).

$$y = (2t+3) * e^{2t} + 3e^{2t}$$

$$= 3e^{2t} + \int_0^t (2(t-v)+3)\, e^{2v}\, dv = 3e^{2t} + \left[\left.(2t - 2v + 3)\tfrac{1}{2}e^{2v}\right|_0^t + \int_0^t e^{2v}\, dv\right]$$

$$= 3e^{2t} - (2t+3)\left(\tfrac{1}{2}\right) + \tfrac{3}{2}e^{2t} + \left.\frac{e^{2v}}{2}\right|_0^t = 5e^{2t} - t - 2$$

3. $\boxed{2}$ Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' - y = g(t), \qquad y(0) = 0, y'(0) = 2.$$

Find the solution. Express your solution in terms of a convolution.

$$s^2 Y - 2 - Y = G(s)$$

$$Y(s^2-1) = G(s) + 2$$

$$Y = G(s)\left(\frac{1}{s^2-1}\right) + \frac{2}{s^2-1}$$

$$y = g(t) * e^t * e^{-t} + e^t * e^{-t}$$

— or —

$$= g(t) * \left(e^t - e^{-t}\right) + e^t - e^{-t}$$

Note: $\frac{2}{s^2-1} = \frac{1}{s-1} - \frac{1}{s+1}$

4. Consider a mass-spring system sitting in front of a cuckoo clock. After π seconds the time is exactly 1 pm. The cuckoo comes out of the clock and strikes the system exerting an impulse on the mass. The system is governed by the symbolic initial value problem

$$x'' + 4x = 2\delta(t - \pi), \qquad x(0) = 0, x'(0) = -2, \tag{1}$$

where $x(t)$ measures the displacement from the equilibrium.

(a) $\boxed{2}$ Determine $x(t)$, i.e. solve the symbolic initial value problem (1).

$$s^2 X + 2 + 4X = 2e^{-\pi s}$$

$$X = e^{-\pi s} \cdot \frac{2}{s^2+4} - \frac{2}{s^2+4}$$

$$x = u(t-\pi)\sin(2t - 2\pi) - \sin 2t$$

$$= u(t-\pi)\sin 2t - \sin 2t$$

(b) $\boxed{1}$ Carefully sketch a graph of $x(t)$ for $t \in [0, 2\pi]$. [HINT: sine is periodic.]