$\qquad$
Due: 31 May 2018

1. 2 For $b>0$, a graph of $y=f(t)$ is given below.


Compute the Laplace transform of $f(t)$.
2. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$
y^{\prime}-2 y=g(t), \quad y(0)=3
$$

(a) 1 Find the solution. Express your solution in terms of a convolution.
(b) 2 If $g(t)=2 t+3$, find the solution by evaluating the convolution integral you found in (a).
3. 2 Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$
y^{\prime \prime}-y=g(t), \quad y(0)=0, y^{\prime}(0)=2 .
$$

Find the solution. Express your solution in terms of a convolution.
4. Consider a mass-spring system sitting in front of a cuckoo clock. After $\pi$ seconds the time is exactly 1 pm . The cuckoo comes out of the clock and strikes the system exerting an impulse on the mass. The system is governed by the symbolic initial value problem

$$
\begin{equation*}
x^{\prime \prime}+4 x=2 \delta(t-\pi), \quad x(0)=0, x^{\prime}(0)=-2, \tag{1}
\end{equation*}
$$

where $x(t)$ measures the displacement from the equilibrium.
(a) 2 Determine $x(t)$, i.e. solve the symbolic initial value problem (1).
(b) 1 Carefully sketch a graph of $x(t)$ for $t \in[0,2 \pi]$.


