Math 274 Homework **Name:** ______________

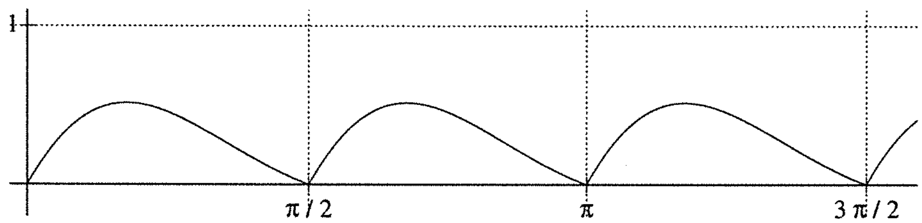
Sections: 7.9, 5.1 Point Values in $\boxed{\text{boxes}}$.

Due: 1 June 2018

1. $\boxed{2}$ Consider the initial value problem

$$x'' + 4x' + 8x = g(t), \quad x(0) = 0, x'(0) = 2.$$

Find $g(t)$ so that the solution $x(t)$ is $\pi/2$-periodic. In particular, a graph of $x(t)$ should look like the following figure.



$x'' + 4x' + 8x = 0$

Taking the Laplace transform gives

$X(s^2 + 4s + 8) = 2$

$X = \frac{2}{(s+2)^2 + 2^2}$

so $x(t) = e^{-2t} \sin 2t$

$x'(t) = -2e^{-2t} \sin 2t + 2e^{-2t} \cos 2t$

$x'(\frac{\pi}{2}) = -2e^{-\pi}$.

so to change the momentum back to 2, we need an impulse of magnitude $2 + 2e^{-\pi}$.

$g(t) = (2 + 2e^{-\pi}) \sum_{n=1}^{\infty} \delta(t - \frac{n\pi}{2})$

2. $\boxed{3}$ Consider the system given by

$$(1) \quad x' = x - 10y + e^{2t}, \qquad x(0) = 3$$
$$y' = x - 5y + \sin 2t, \qquad y(0) = 7.$$

Convert the system into a second order initial value problem in y. **Do not solve the initial value problem.**

$x = y' + 5y - \sin 2t$

$x' = y'' + 5y' - 2\cos 2t$

Substituting into (1) gives

$y'' + 5y' - 2\cos 2t = y' + 5y - \sin 2t - 10y + e^{2t}$

so $y'' + 4y' + 5y = 2\cos 2t - \sin 2t + e^{2t}$

$y(0) = 7$, $y'(0) = -32$

3. Applying the substitution

$$x = y' + y \tag{1}$$

to the symbolic system of equations

$$x' = x - 10y + 3\delta(t-1), \qquad x(0) = 1$$
$$y' = x - y, \qquad y(0) = 1$$

converts the system into the symbolic initial value problem

$$y'' + 9y = 3\delta(t-1), \qquad y(0) = 1, y'(0) = 0. \tag{2}$$

(a) $\boxed{3}$ Solve the initial value problem (2) for $y(t)$. Express your solution as a piecewise defined function.

$$s^2 Y - s + 9Y = 3e^{-s}$$

$$Y(s^2+9) = 3e^{-s} + s$$

$$Y = e^{-s}\frac{3}{s^2+9} + \frac{s}{s^2+9}$$

$$y = u(t-1)\sin(3t-3) + \cos 3t$$

$$y = \begin{cases} \cos 3t & t < 1 \\ \cos 3t + \sin(3t-3) & t > 1 \end{cases} \qquad \text{so } y' = \begin{cases} -3\sin 3t & t < 1 \\ -3\sin 3t + 3\cos(3t-3) & t > 1 \end{cases}$$

(b) $\boxed{2}$ Using the substitution (1), find $x(t)$ for $t \in (0,1)$ and $t \in (1,\infty)$.

$$x = \begin{cases} \cos 3t - 3\sin 3t & t < 1 \\ \cos 3t - 3\sin 3t + \sin(3t-3) + 3\cos(3t-3) & t > 1 \end{cases}$$