Math 274 Quiz 1 **Name:** ____________________

Sections: 1.1-1.3, Phase Line

15 May 2018

Point values in $\boxed{\text{boxes}}$.

1. Consider the initial value problem

$$\frac{dy}{dx} = -y^2, \qquad y(1) = 2. \tag{1}$$

(a) $\boxed{2}$ Verify $y(x) = \dfrac{1}{x+c}$, where c is an arbitrary constant, is a one-parameter family of solutions to $\dfrac{dy}{dx} = -y^2$.

$$\frac{dy}{dx} = \frac{-1}{(x+C)^2} = -\left(\frac{1}{x+c}\right)^2 = -y^2$$

(b) $\boxed{2}$ Does Theorem 1 (Existence/Uniqueness) apply to the initial value problem (1)? Why, or why not?

Yes, $-y^2$ & $\frac{\partial}{\partial y}(-y^2) = -2y$ are both continuous for all y.

(c) $\boxed{2}$ Solve the initial value problem (1), i.e. find c so that $y(x) = \dfrac{1}{x+c}$ satisfies the initial data[1].

$$y(1) = \frac{1}{1+C} = 2$$

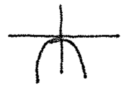
so $\frac{1}{2} = 1 + C$ so $C = -\frac{1}{2}$.

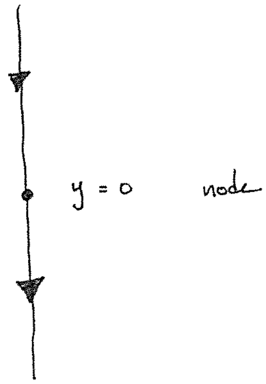
The solution to the ivp is then $y = \dfrac{1}{x - \frac{1}{2}}$

[1]The solution to an initial value problem is not a number, so your solution should not be of the form $c = 7$ with a box around it. What type of thing is a solution to an intial value problem?

2. $\boxed{4}$ For each of the following equations, label the equilibria and classify each as a sink, source, or node on the phase line. Include direction arrows. Additionally, for each, predict the asymptotic behavior as $t \to \infty$ of the solution satisfying $y(0) = -0.5$.

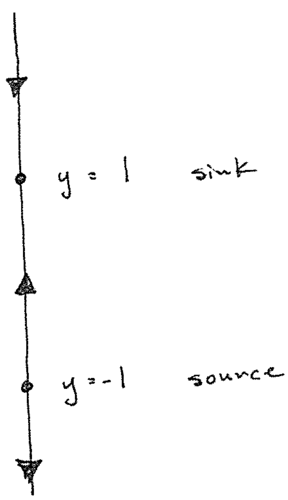
(a) $\dfrac{dy}{dt} = -y^2$





$y \longrightarrow -\infty$ as $t \longrightarrow \infty$

(b) $\dfrac{dy}{dt} = 1 - y^2$



$y \longrightarrow 1$ as $t \longrightarrow \infty$