Math 274 Quiz 10 **Name:** ______________________

Section: 9.4

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Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ The vectors

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 10 \\ 0 \\ 6 \end{bmatrix}$$

satisfy

$$2\mathbf{x}_1 + 4\mathbf{x}_2 - \mathbf{x}_3 = \mathbf{0}.$$

The vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are linearly **independent/dependent** on $(-\infty, \infty)$. (Circle one.)

2. $\boxed{1}$ The vectors

$$\mathbf{x}_4 = \begin{bmatrix} 2e^t \\ 2 \end{bmatrix}, \mathbf{x}_5 = \begin{bmatrix} te^t \\ t \end{bmatrix}$$

satisfy

$$t\mathbf{x}_4 - 2\mathbf{x}_5 = \mathbf{0} \text{ for all } t.$$

The vectors $\mathbf{x}_4, \mathbf{x}_5$ are linearly **independent/dependent** on $(-\infty, \infty)$. (Circle one.)

CONTINUED ON THE REVERSE.

3. Let

$$\mathbf{A} = \begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix}, \mathbf{x}_1 = \begin{bmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} \sin 2t + 2\cos 2t \\ \sin 2t \end{bmatrix}$$

and consider the equation

$$\mathbf{x}' = \mathbf{A}\mathbf{x}. \tag{1}$$

(a) $\boxed{3}$ The vector function $\mathbf{x}_1$ is a solution to (1), verify $\mathbf{x}_2$ is also a solution to (1).

$$\vec{x}_2{}' = \begin{bmatrix} 2\cos 2t - 4\sin 2t \\ 2\cos 2t \end{bmatrix} \quad = \checkmark$$

$$A\vec{x}_2 = \begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2\cos 2t + \sin 2t \\ \sin 2t \end{bmatrix} = \begin{bmatrix} 2\cos 2t + \sin 2t - 5\sin 2t \\ 2\cos 2t + \sin 2t - \sin 2t \end{bmatrix}$$

(b) $\boxed{2}$ Verify $\{\mathbf{x}_1, \mathbf{x}_2\}$ is a fundamental solution set for (1).

$$W[\vec{x}_1, \vec{x}_2] = \begin{vmatrix} \cos 2t - 2\sin 2t & \sin 2t + 2\cos 2t \\ \cos 2t & \sin 2t \end{vmatrix} = \cos 2t \sin 2t - 2\sin^2 2t - \cos 2t \sin 2t - 2\cos^2 2t$$

$$= -2(\sin^2 2t + \cos^2 2t) = -2 \neq 0$$

for all t.

(c) $\boxed{1}$ Find a fundamental matrix for (1) and evaluate it at $t = 0$, i.e., find $\mathbf{X}(0)$.

$$\underline{X}(0) = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\underline{X}^{-1}(0) = -\frac{1}{2}\begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}$$

(d) $\boxed{2}$ Find the solution to the initial value problem

$$\mathbf{x}' = \mathbf{A}\mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

You may find it useful to know that if $\mathbf{X} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible then $\mathbf{X}^{-1} = \frac{1}{|\mathbf{X}|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\vec{x} = \underline{X}\,\underline{X}^{-1}(0)\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \underline{X}\left(-\frac{1}{2}\begin{bmatrix} -2 \\ -2 \end{bmatrix}\right) = \underline{X}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cos 2t - \sin 2t \\ \cos 2t + \sin 2t \end{bmatrix}$$