Math 274 Quiz 11 **Name:** ____________________

Sections: 9.5,9.6

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Point values in $\boxed{\text{boxes}}$.

1. Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}.$$

(a) $\boxed{5}$ Find the eigenvalues and corresponding eigenvectors for $\mathbf{A}$.

$r^2 - 3r - 4 = 0$

$(r-4)(r+1)$

$r = 4$

$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \vec{0}$ $\quad\quad 4, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$r = -1$

$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \vec{0}$ $\quad\quad -1, \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\boxed{2}$ Find a general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

$$\vec{x} = C_1 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-t}\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

2. $\boxed{3}$ The real valued matrix $\mathbf{B}$ has an eigenvalue $r = 1 + 2i$ with a corresponding eigenvector $\mathbf{u} = \begin{bmatrix} 1+2i \\ 3 \end{bmatrix}$. Find a general solution to $\mathbf{x}' = \mathbf{B}\mathbf{x}$.

$$\vec{x} = C_1 e^t \left(\cos 2t \begin{bmatrix} 1 \\ 3 \end{bmatrix} - \sin 2t \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) + C_2 e^t \left(\sin 2t \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \cos 2t \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$$

$$= C e^t \begin{bmatrix} \cos 2t - 2\sin 2t \\ 3\cos 2t \end{bmatrix} + C_2 e^t \begin{bmatrix} \sin 2t + 2\cos 2t \\ 3\sin 2t \end{bmatrix}$$