Math 274 Quiz 4 **Name:** ____________________

Sections: 4.4,4.5

22 May 2018

Point values in $\boxed{\text{boxes}}$.

Method of Undetermined Coefficients

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{rt}$$

where $P_m(t)$ is a polynomial of degree m, use the form

$$y_p(t) = t^s\left(A_m t^m + \cdots + A_1 t + A_0\right)e^{rt};$$

if r is not a root of the associated auxiliary equation, take $s = 0$; if r is a simple root, take $s = 1$; and if r is a double root, take $s = 2$.

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{\alpha t}\cos\beta t + Q_n(t)e^{\alpha t}\sin\beta t$$

where $P_m(t)$ and $Q_n(t)$ are polynomials of degree m and n, respectively, use the form

$$y_p(t) = t^s\left(A_k t^k + \cdots + A_1 t + A_0\right)e^{\alpha t}\cos\beta t + t^s\left(B_k t^k + \cdots + B_1 t + B_0\right)e^{\alpha t}\sin\beta t;$$

where k is the larger of m and n. If $\alpha + i\beta$ is not a root of the associated auxiliary equation, take $s = 0$; if so take $s = 1$.

1. $\boxed{4}$ Find the appropriate form using the Method of Undetermined Coefficients for a particular solution to the following. **Do not** solve for the unknown constants.

   (a) $y'' - 2y' + y = (3t+2)e^{3t}$

   $r^2 - 2r + 1 = (r-1)^2 = 0$

   $r = 1$ is a double root

   $y_p = (At + B)e^{3t}$

   (b) $y'' - 2y' + y = 8 - 4e^t$

   $y_p = A + Bt^2e^t$

   (c) $y'' - 2y' + y = 3t\sin t$

   $y_p = (At+B)\cos t + (Ct+D)\sin t$

   (d) $y'' - 2y' + y = e^t\cos 3t$

   $y_p = Ae^t\cos 3t + Be^t\sin 3t$

2. $\boxed{1}$ Find the form of a particular solution to $y'' + y = \sin(.9t)$. **Do not** solve for the unknown constants. Do solutions of this form stay bounded as $t \to \infty$.

   $r^2 + 1 = 0$ so $r = \pm i$

   $y_p = A\sin(.9t) + B\cos(.9t)$, yes, bounded (by $\pm\sqrt{A^2+B^2}$)

3. $\boxed{1}$ Find the form of a particular solution to $y'' + y = \sin(t)$. **Do not** solve for the unknown constants. Do solutions of this form stay bounded as $t \to \infty$.

   $y_p = At\sin t + Bt\cos t$, unbounded if A or $B \neq 0$

4. $\boxed{4}$ Find a general solution for the following.

$$y'' - y = 7e^{2t} - t^2$$

$r^2 - 1 = 0 \qquad r = \pm 1 \qquad y = C_1 e^t + C_2 e^{-t}$ solves the homogeneous

try $y_p = Ae^{2t} + Bt^2 + Ct + D$

$y_p' = 2Ae^{2t} + 2Bt + C$

$y_p'' = 4Ae^{2t} + 2B$

substituting gives

$y_p'' - y_p = 4Ae^{2t} + 2B - Ae^{2t} - Bt^2 - Ct - D = 7e^{2t} - t^2$

$e^{2t}: \quad 4A - A = 7 \quad$ so $\quad A = 7/3$

$t^2: \quad -B = -1 \quad$ so $\quad B = 1$

$t: \quad -C = 0 \quad$ so $\quad C = 0$

$t^0: \quad 2B - D = 0 \quad$ so $\quad D = 2$

A general solution is then

$$y = C_1 e^t + C_2 e^{-t} + \frac{7}{3} e^{2t} + t^2 + 2$$