Math 274 Quiz 5 **Name:** ______________________

Sections: 4.6, 4.7

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Point values in $\boxed{\text{boxes}}$.

Variation of Parameters

If y_1 and y_2 are linearly independent solutions to $y'' + p(t)y' + q(t)y = 0$, then a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is given by

$$y_p(t) = y_1(t) \int \frac{-g(t)y_2(t)}{W[y_1, y_2](t)}\,dt + y_2(t) \int \frac{g(t)y_1(t)}{W[y_1, y_2](t)}\,dt.$$

1. For $x > 0$, consider the differential equation

$$xy'' - y' + (1-x)y = \frac{\sin x}{2x}.$$

(a) $\boxed{4}$ Both $y_1 = e^x$ and $y_2 = e^{-x}(2x+1)$ are solutions to the associated homogeneous equation, show that they are linearly independent.

$$W\left[e^x, e^{-x}(2x+1)\right] = \begin{vmatrix} e^x & e^{-x}(2x+1) \\ e^x & -e^{-x}(2x+1) + e^{-x}(2) \end{vmatrix}$$

$$= -(2x+1) + 2 - (2x+1)$$

$$= -4x \neq 0 \quad \text{for } x > 0$$

(b) $\boxed{2}$ Set up the variation of parameters expression for the particular solution to the original inhomogeneous equation. **DO NOT EVALUATE.**

Std form: $y'' - \frac{1}{x}y' + \frac{(1-x)}{x}y = \frac{\sin x}{2x^2}$, so

$$y_p = e^x \int \frac{(-\sin x)\,e^{-x}(2x+1)}{2x^2\,(-4x)}\,dx + e^{-x}(2x+1) \int \frac{\sin x\,(e^x)}{2x^2\,(-4x)}\,dx$$

Reduction of Order

If $y_1(t)$ is a solution, not identically zero, to $y'' + p(t)y' + q(t)y = 0$ on I, then

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t)\,dt}}{(y_1(t))^2}\,dt$$

is a second, linearly independent solution.

2. $\boxed{4}$ Find the general solution to the equation

$$2t^2y'' + ty' - 3y = 0.$$

Note that $y_1 = t^{-1}$ is a solution to this equation.

Std Form: $y'' + \frac{1}{2t}y' - \frac{3}{2t^2}y = 0$

$$y_2 = t^{-1} \int \frac{e^{-\int \frac{1}{2}\cdot\frac{1}{t}\,dt}}{t^{-2}}\,dt$$

$$= t^{-1} \int t^2 \cdot e^{-\frac{1}{2}\ln|t|}\,dt$$

$$= t^{-1} \int t^2 \cdot t^{-1/2}\,dt$$

$$= t^{-1} \int t^{3/2}\,dt$$

$$= t^{-1} \cdot \frac{2}{5} t^{5/2}$$

$$= \frac{2}{5} t^{3/2} \quad \text{OR} \quad t^{3/2}$$

A general solution is then

$$y = C_1 t^{-1} + C_2 t^{3/2}$$