Math 274 Quiz 5 **Name:** ____________________

Sections: 4.6, 4.7 Point values in $\boxed{\text{boxes}}$.

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Variation of Parameters

If y_1 and y_2 are linearly independent solutions to $y'' + p(t)y' + q(t)y = 0$, then a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is given by

$$y_p(t) = y_1(t) \int \frac{-g(t)y_2(t)}{W[y_1, y_2](t)}\, dt + y_2(t) \int \frac{g(t)y_1(t)}{W[y_1, y_2](t)}\, dt.$$

1. For $x > 0$, consider the differential equation

$$xy'' - y' + (1 - x)y = \frac{\sin x}{2x}.$$

   (a) $\boxed{4}$ Both $y_1 = e^x$ and $y_2 = e^{-x}(2x+1)$ are solutions to the associated homogeneous equation, show that they are linearly independent.

   (b) $\boxed{2}$ Set up the variation of parameters expression for the particular solution to the original inhomogeneous equation. **DO NOT EVALUATE**.

Reduction of Order

If $y_1(t)$ is a solution, not identically zero, to $y'' + p(t)y' + q(t)y = 0$ on I, then

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t)\,dt}}{(y_1(t))^2}\,dt$$

is a second, linearly independent solution.

2. $\boxed{4}$ Find the general solution to the equation

$$2t^2y'' + ty' - 3y = 0.$$

Note that $y_1 = t^{-1}$ is a solution to this equation.