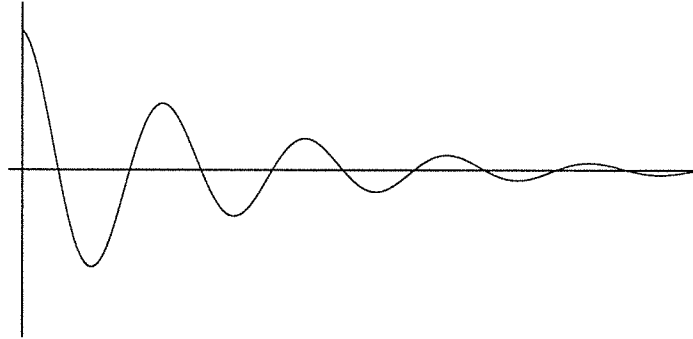


1. For $x > 0$, consider the mass spring system

$$mx'' + bx' + kx = 0.$$

(a) 3 Let $x(t)$ be a solution to this system, and the graph of $x(t)$ below.



Is the quantity $b^2 - 4mk$: Negative, Positive, Zero (circle one).

(b) 3 Which of the functions below could be possible solutions to the system given (a):

• $x(t) = 3e^{-t} \cos(4t),$

• $x(t) = 4 \sin(t) + 5 \cos(t),$

• $x(t) = \frac{1}{2}e^{-2t} + \frac{1}{2}te^{-2t}.$

2. 4 Recall that *hyperbolic cosine* can be defined by

$$\cosh u = \frac{e^u + e^{-u}}{2}.$$

For a a constant, show that

$$\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2 - a^2}.$$

$$\cosh(at) = \frac{1}{2} (e^{at} + e^{-at})$$

$$\mathcal{L}\{\cosh(at)\} = \frac{1}{2} \mathcal{L}\{e^{at} + e^{-at}\}$$

$$= \frac{1}{2} \left[\frac{1}{s-a} + \frac{1}{s+a} \right]$$

$$= \frac{1}{2} \left[\frac{s+a + s-a}{s^2 - a^2} \right]$$

$$= \frac{s}{s^2 - a^2}$$