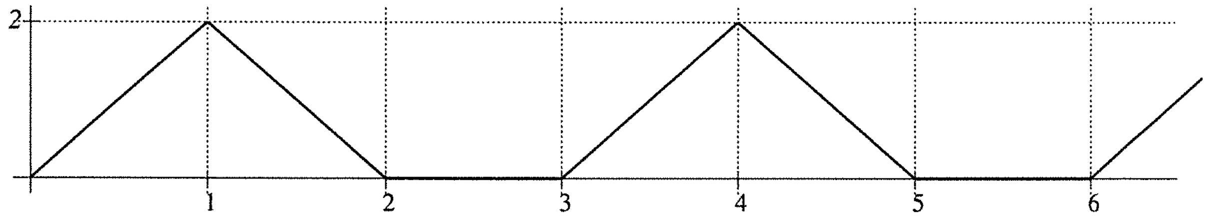
Math 274 Quiz 8 **Name:** ______________________

Sections: 7.6 Point values in $\boxed{\text{boxes}}$.

30 May 2018

1. $\boxed{3}$ A graph of $y = f(t)$ is given below.



Find the Laplace transform of $f(t)$.

$$f_3 = 2t + u(t-1)\left[-2t + 4 - 2t\right] - u(t-2)\left[-2t+4\right]$$

$$= 2t + u(t-1)\left(-4(t-1)\right) - u(t-2)\left(-2(t-2)\right)$$

$$F_3 = \frac{2}{s^2} + e^{-s}\left(\frac{-4}{s^2}\right) + \frac{2}{s^2}e^{-2s}$$

$$= \frac{2 - 4e^{-s} + 2e^{-2s}}{s^2}$$

$$F = \frac{2 - 4e^{-s} + 2e^{-2s}}{s^2\left(1-e^{-3s}\right)}$$

2. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' + y' = g(t), \qquad y(0) = 0, y'(0) = 0.$$

(a) $\boxed{3}$ Find the solution. Express your solution in terms of a convolution.

$$s^2 Y + sY = G \qquad \frac{1}{s^2+s} = \frac{1}{s} - \frac{1}{s+1}$$

$$Y(s^2+s) = G$$

$$Y = \frac{G}{s^2+s} = G(s)\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$y = g(t) * (1 - e^{-t})$$

(b) $\boxed{1}$ Express the solution in terms of the appropriate convolution integral. **Do not evaluate the integral.**

$$y = \int_0^t g(t-v)(1-e^{-v})\,dv$$

(c) $\boxed{3}$ If $g(t) = e^{-2t}$, find the solution, i.e., evaluate the convolution.

$$y = \int_0^t e^{-2(t-v)}(1-e^{-v})\,dv$$

$$= e^{-2t}\int_0^t (e^{2v} - e^{v})\,dv = e^{-2t}\left(\frac{1}{2}e^{2v} - e^{v}\right)\Big|_0^t$$

$$= e^{-2t}\left(\frac{1}{2}e^{2t} - e^{t} + \frac{1}{2}\right) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$