Math 274 Quiz 9
Sections: 7.6
30 May 2018

Name: ____________________

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Consider the system given by

$$x' = 2x - 5y + 7, \qquad x(0) = 5$$
$$y' = x + 4y + e^{t^2}, \qquad y(0) = 1.$$

Convert the system into a second order initial value problem in y. **Do not solve.**

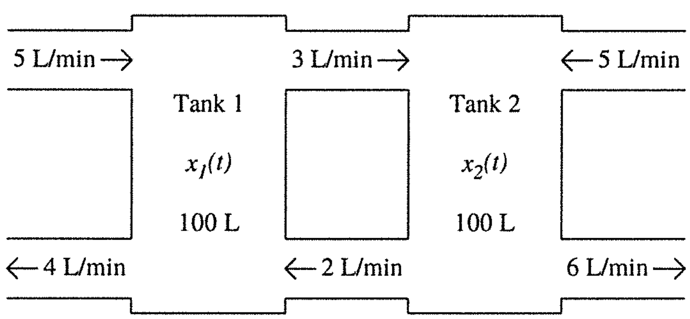
$x = y' - 4y - e^{t^2}$

$x' = y'' - 4y' - 2te^{t^2}$

so $\quad y'' - 4y' - 2te^{t^2} = 2(y' - 4y - e^{t^2}) - 5y + 7$

$y'' - 6y' + 13y = e^{t^2}(2t - 2) + 7 \qquad y(0) = 1 \qquad y'(0) = 10$

2. $\boxed{2}$ Tank 1 initially contains 100 L of a brine mixture with concentration 0.2 kg/L of salt. Tank 2 initially contains 100 L of a brine mixture with concentration 0.4 kg/L of salt. Both tanks are well mixed. A mixture containing 0.3 kg/L of salt is flowing into each tank at the rate specified in the figure. Similarly, the figure shows the rate the mixtures are flowing between each tank and being drained. Let $x(t)$ be the amount of salt in tank 1 in kg, and $y(t)$ be the amount of salt in tank 2 in kg.



Set up a system of first order equations to model the amount of salt in each tank. Include initial data. **Do not convert to second order nor solve.**

$x' = \frac{-7x}{100} + \frac{2y}{100} + 1.5 \qquad x(0) = 20$

$y' = \frac{3x}{100} - \frac{8y}{100} + 1.5 \qquad y(0) = 40$

3. $\boxed{4}$ Consider the mass-spring system given by the symbolic initial value problem

$$y'' + y = \sqrt{3}\delta(t - \pi/2), \qquad y(0) = 0, y'(0) = 1. \tag{1}$$

(a) Find the solution to (1).

$$s^2 Y - 1 + Y = \sqrt{3}\, e^{-\pi s/2}$$

$$Y = \frac{1}{s^2+1} + \sqrt{3}\, e^{-\pi s/2} \cdot \frac{1}{s^2+1}$$

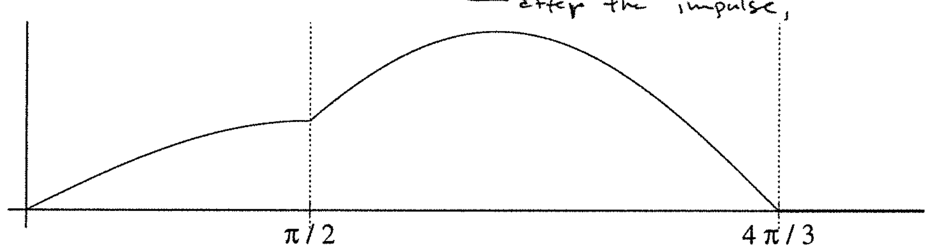
$$y = \sin t + \sqrt{3}\, u\left(t - \tfrac{\pi}{2}\right) \sin\left(t - \tfrac{\pi}{2}\right)$$

$$y = \begin{cases} \sin t & t < \frac{\pi}{2} \\ \sin t + \sqrt{3} \sin\left(t - \frac{\pi}{2}\right) & t > \frac{\pi}{2} \end{cases}$$

$$y' = \begin{cases} \cos t & t < \frac{\pi}{2} \\ \cos t + \sqrt{3} \cos\left(t - \frac{\pi}{2}\right) & t > \frac{\pi}{2} \end{cases} \qquad y'\left(\tfrac{4\pi}{3}\right) = -\tfrac{1}{2} + \sqrt{3}\left(\tfrac{-\sqrt{3}}{2}\right) = -2$$

(b) $\boxed{1}$ Find the magnitude of the impulse needed to stop the motion of the system when it first returns to equilibrium at time $t_1 = 4\pi/3$, i.e., the solution has the following graph.

after the impulse,



The magnitude is 2, OR

$$y'' + y = \sqrt{3}\,\delta(t - \pi/2) + 2\,\delta(t - 4\pi/3), \qquad y(0) = 0,\ y'(0) = 1$$

has the specified solution graph.