Name: $\qquad$
Sections: 7.6
Point values in boxes.
30 May 2018

1. 3 Consider the system given by

$$
\begin{array}{ll}
x^{\prime}=2 x-5 y+7, & x(0)=5 \\
y^{\prime}=x+4 y+e^{t^{2}}, & y(0)=1 .
\end{array}
$$

Convert the system into a second order initial value problem in standard form in $y$. Do not solve.
2. 2 Tank 1 initially contains 100 L of a brine mixture with concentration $0.2 \mathrm{~kg} / \mathrm{L}$ of salt. Tank 2 initially contains 100 L of a brine mixture with concentration $0.4 \mathrm{~kg} / \mathrm{L}$ of salt. Both tanks are well mixed. A mixture containing $0.3 \mathrm{~kg} / \mathrm{L}$ of salt is flowing into each tank at the rate specified in the figure. Similarly, the figure shows the rate the mixtures are flowing between each tank and being drained. Let $x(t)$ be the amount of salt in tank 1 in kg , and $y(t)$ be the amount of salt in tank 2 in kg.


Set up a system of first order equations to model the amount of salt in each tank. Include initial data. Do not convert to second order nor solve.
3. 4 Consider the mass-spring system given by the symbolic initial value problem

$$
\begin{equation*}
y^{\prime \prime}+y=\sqrt{3} \delta(t-\pi / 2), \quad y(0)=0, y^{\prime}(0)=1 \tag{1}
\end{equation*}
$$

(a) Find the solution to (1). Express your solution as a piecewise defined function.
(b) 1 Find the magnitude of the impulse needed to stop the motion of the system when it first returns to equilibrium at time $t_{1}=4 \pi / 3$, i.e., after the impulse at $t_{1}=4 \pi / 3$ the solution has the following graph.


