Math 274 In-Class **Worksheet 10p**
Section: 7.8

1. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' + 9y = g(t), \qquad y(0) = 1, y'(0) = 6.$$

Find the solution. Express your solution in terms of a convolution.

2. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' - 3y' + 2y = g(t), \qquad y(0) = 1, y'(0) = 1.$$

(a) Find the solution. Express your solution in terms of a convolution.

(b) Express the convolution in (a) as an appropriate integral.

(c) If $g(t) = e^t$, evaluate the convolution.

3. Let $F(s) = \dfrac{5}{s^2 - s - 6}$.

   (a) Use partial fractions to find the inverse Laplace transform $f(t)$.

   (b) Use the convolution theorem to find the inverse Laplace transform $f(t)$.

   (c) Compute the convolution integral to show your solutions are equivalent.

4. Compute the inverse Laplace transform of $F(s) = \dfrac{1}{(s^2 + 4)^2}$.

   Note: $\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$ is a useful identity.