1. Consider the mass-spring system given by the initial value problem

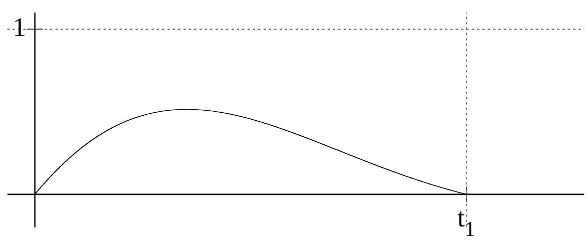
$$x'' + 2x' + 5x = 0, \qquad x(0) = 0, x'(0) = 2. \tag{1}$$

(a) Find the solution to (1).

(b) Find the magnitude of the impulse needed to stop the motion of the system when it first returns to equilibrium at time t_1, i.e., find M so that the solution to the symbolic initial value problem

$$x'' + 2x' + 5x = M\delta(t - \pi/2), \qquad x(0) = 0, x'(0) = 2$$

has the following graph.



You may find the following useful.

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

2. Find the solution to the symbolic initial value problem

$$y'' + 2\pi y' + 5\pi^2 y = 4\pi\delta(t-1), \qquad x(0) = 0, x'(0) = 2\pi.$$

3. Use scratch paper, and your remaining time to investigate the following.

   (a) For $n > 0$, consider the initial value problem

   $$y'' + y = n\left(1 - u(t - 1/n)\right), \qquad y(0) = y'(0) = 0.$$

   Find the solution, $y_n(t)$, and express it as a piecewise defined function that depends on n.

   (b) Evaluate

   $$\lim_{n\to\infty} y_n(t).$$

   (c) Solve the initial value problem

   $$y'' + y = \delta(t), \qquad y(0) = y'(0) = 0.$$

   (d) Solve the initial value problem

   $$y'' + y = 0, \qquad y(0) = 0, y'(0) = 1.$$

   (e) What do you notice about the solutions to (b), (c), and (d)? Is it what you expected?