Math 274 In-Class **Worksheet 1a**

Sections: 1.1-1.2

1. Show $\phi(x) = \dfrac{\sin x + c}{x^2}$, where c is an arbitrary constant, is a one-parameter family of solutions to

$$x^2y' + 2xy = \cos x.$$

$$\varphi' = \frac{x^2 \cos x - (\sin x + c)(2x)}{x^4}$$

$$x^2 \varphi' + 2x\varphi = \frac{x^2 \cos x - 2x(\sin x + c)}{x^2} + \frac{2x(\sin x + c)}{x^2}$$

$$= \frac{x^2 \cos x}{x^2}$$

$$= \cos x \quad \checkmark$$

2. Find c so that $\phi(x) = \dfrac{\sin x + c}{x^2}$ solves the initial value problem

$$x^2y' + 2xy = \cos x, \qquad y(1) = 0.$$

$\varphi(1) = \dfrac{\sin 1 + c}{1} = 0$ so $c = -\sin 1$,

i.e. $\varphi(x) = \dfrac{\sin x - \sin 1}{x^2}$ solves the ivp.

3. Show $y^2 + \cos(xy) - e^x = 7$ is an implicit solution to

$$y' = \frac{e^x + y\sin(xy)}{2y - x\sin(xy)}.$$

We differentiate implicitly to see

$$2y\,y' - \sin(xy)\left(y + xy'\right) - e^x = 0$$

$$2y\,y' - x\sin(xy)\,y' = e^x + y\sin(xy)$$

so

$$y' = \frac{e^x + y\sin xy}{2y - x\sin xy}$$

4. For each of the following initial value problems, determine if Theorem 1 implies that a unique solution exists.

(a) **Yes** / **No:** $y' = -\sqrt{y}$, $y(2) = 1$
(b) **Yes** / **No:** $y' = -\sqrt{y}$, $y(2) = 0$
(c) **Yes** / **No:** $xy' = \cos y$, $y(0) = 1$
(d) **Yes** / **No:** $xy' = \cos y$, $y(2) = 0$

5. A simplified hurricane wind intensity model is given by

$$\frac{dU}{dt} = U(1 - U^n)$$

where U is the nondimensional wind speed and n is a positive constant that measures how close the wind intensity can come to the maximum potential intensity.

For today we further simplify the model by letting $n = 2$ resulting in the equation

$$\frac{dU}{dt} = U(1 - U^2). \tag{1}$$

Verify that

$$U(t) = U_0 e^t \left(1 + U_0^2(e^{2t} - 1)\right)^{-1/2} \tag{2}$$

solves equation (1) with initial condition $U(0) = U_0$. In other words,

(a) verify (2) satisfies $U(0) = U_0$, and

$$U(0) = U_0 \left(1 + U_0^2(1-1)\right)^{-1/2} = U_0 \quad \checkmark$$

(b) verify that (2) satisfies (1).

$$\frac{dU}{dt} = \underbrace{U_0 e^t \left(1 + U_0^2(e^{2t} - 1)\right)^{-1/2}}_{U} - \frac{1}{2} U_0 e^t \left(1 + U_0^2(e^{2t} - 1)\right)^{-3/2} \left(2U_0^2 e^{2t}\right)$$

$$= U - U_0^3 e^{3t} \left(1 + U_0^2(e^{2t} - 1)\right)^{-3/2}$$

$$= U - \left(U_0 e^t \left(1 + U_0^2(e^{2t} - 1)\right)^{-1/2}\right)^3$$

$$= U - U^3$$

$$= U(1 - U^2) \quad \checkmark$$