1. Show $\phi(x) = \dfrac{\sin x + c}{x^2}$, where c is an arbitrary constant, is a one-parameter family of solutions to

$$x^2 y' + 2xy = \cos x.$$

2. Find c so that $\phi(x) = \dfrac{\sin x + c}{x^2}$ solves the initial value problem

$$x^2 y' + 2xy = \cos x, \qquad y(1) = 0.$$

3. Show $y^2 + \cos(xy) - e^x = 7$ is an implicit solution to

$$y' = \frac{e^x + y \sin(xy)}{2y - x \sin(xy)}.$$

4. For each of the following initial value problems, determine if Theorem 1 implies that a unique solution exists.

(a) **Yes / No:** $y' = -\sqrt{y}$, $y(2) = 1$

(b) **Yes / No:** $y' = -\sqrt{y}$, $y(2) = 0$

(c) **Yes / No:** $xy' = \cos y$, $y(0) = 1$

(d) **Yes / No:** $xy' = \cos y$, $y(2) = 0$

5. A simplified hurricane wind intensity model is given by

$$\frac{dU}{dt} = U(1 - U^n)$$

where U is the nondimensional wind speed and n is a positive constant that measures how close the wind intensity can come to the maximum potential intensity.

For today we further simplify the model by letting $n = 2$ resulting in the equation

$$\frac{dU}{dt} = U(1 - U^2). \tag{1}$$

Verify that

$$U(t) = U_0 e^t \left(1 + U_0^2(e^{2t} - 1)\right)^{-1/2} \tag{2}$$

solves equation (1) with initial condition $U(0) = U_0$. In other words,

(a) verify (2) satisfies $U(0) = U_0$, and

(b) verify that (2) satisfies (1).