Math 274 In-Class **Worksheet 1p**

Sections: 1.3, Phase Line

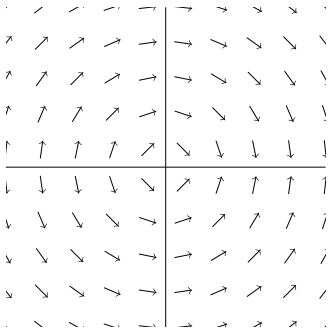
1. Consider the autonomous[1] differential equation $\dfrac{dp}{dt} = \dfrac{p(4-p)}{4}$.

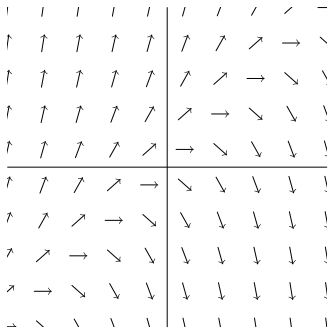
   (a) Sketch the isoclines passing through $(0,-2)$, $(0,0)$, $(0,2)$, $(0,4)$, and $(0,6)$. Include direction arrows and label each isocline with the appropriate slope.

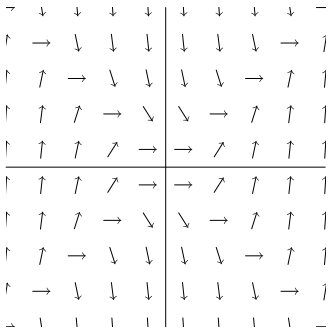
   (b) Sketch the solution curves satisfying $p(0) = -2$, $p(0) = 2$, $p(0) = 4$, and $p(0) = 6$.

2. Match each direction field below with the differential equation it corresponds to from the following list. Use the space below each to write the letter (A, B, or C) corresponding.

   A. $\dfrac{dy}{dx} = x^2 - y^2$ B. $\dfrac{dy}{dx} = \dfrac{-x}{y}$ C. $\dfrac{dy}{dx} = y - x$







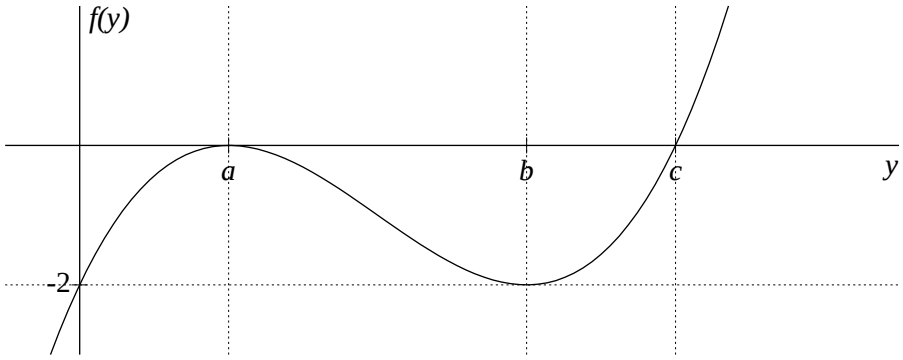
   Equation _____ Equation _____ Equation _____

3. Each of the above direction fields shows a window of $[-4,4] \times [-4,4]$. On each, sketch the solution curve satisfying $y(0) = 2$.

[1] Autonomous means there is no explicit dependence on the independent variable on the right hand side.

4. Sketch the phase line for $\dfrac{dp}{dt} = \dfrac{p(4-p)}{4}$. Identify each equilibria as a sink, source, or node. (Sketch your phase line horizontally for convenience with $-\infty$ on the left and $+\infty$ on the right, as usual.)

5. The graph of $f(y)$ is given below, use it to answer the following.



(a) Sketch the phase line for $y' = f(y)$. Identify each equilibria as a sink, source, or node.

(b) For what values of k does $y' = f(y) + k$ have three equilibria? Sketch a representative phase line for one of those k values[2] including the standard arrows and labels.

(c) For what values of k does $y' = f(y) + k$ have one equilibrium? Sketch a representative phase line for one of those k values[3] including the standard arrows and labels.

(d) For what values of $k \neq 0$ does $y' = f(y) + k$ have two equilibria? Sketch the phase line for that k value, including the standard arrows and label.

[2] Obviously we don't know the y values; use $m < n < q$ for labels.
[3] Obviously we don't know the y value; use j for the label.