

1. Find an implicit solution to

$$\frac{dy}{dt} = ty^2 + t, \quad y(0) = 1.$$

$$\int \frac{dy}{y^2+1} = \int t \, dt$$

$$\arctan y = \frac{t^2}{2} + C$$

$$\frac{\pi}{4} = C$$

$$\text{so } \arctan y = \frac{t^2}{2} + \frac{\pi}{4}$$

2. Consider

$$\frac{dy}{dx} = \frac{xe^x(y^2 - 4)}{4}. \quad (1)$$

- (a) Find an implicit general solution to (1).

$$\int \frac{4}{y^2-4} dy = \int x e^x dx \quad \text{or } y=2 \quad \text{or } y=-2$$

$$\text{Note: } \frac{4}{y^2-4} = \frac{1}{y-2} - \frac{1}{y+2}$$

$$\int \left( \frac{1}{y-2} - \frac{1}{y+2} \right) dy = \int x e^x dx$$

$$\ln|y-2| - \ln|y+2| = x e^x - e^x + C \quad \text{or } y=2 \quad \text{or } y=-2$$

- (b) Find an implicit solution to (1) satisfying  $y(0) = 1$ .

$$\ln|-1| - \ln|3| = -1 + C \quad \text{so } C = 1 - \ln 3$$

so

$$\ln \left| \frac{y-2}{y+2} \right| = e^x (x-1) + 1 - \ln 3$$

- (c) Find a solution to (1) satisfying  $y(0) = 2$ .

$$y = 2$$

3. Work # 31 in your text. It is an important example that we will want to compare to this afternoon during our discussion of Section 2.3.

$$\frac{dy}{dx} = x y^3$$

$$\int y^{-3} dy = \int x dx$$

$$-\frac{y^{-2}}{2} = \frac{x^2}{2} + C$$

$$y^{-2} = C - x^2$$

$$y = \pm \frac{1}{\sqrt{C - x^2}}$$

$$y(0) = 1 \Rightarrow y = \frac{1}{\sqrt{1 - x^2}} \quad \text{for } x \in (-1, 1)$$

$$y(0) = \frac{1}{2} \Rightarrow y = \frac{1}{\sqrt{4 - x^2}} \quad \text{for } x \in (-2, 2)$$

$$y(0) = 2 \Rightarrow y = \frac{1}{\sqrt{\frac{1}{4} - x^2}} \quad \text{for } x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$y(0) = a > 0 \Rightarrow y = \frac{1}{\sqrt{\frac{1}{a^2} - x^2}} \quad \text{for } x \in \left(-\frac{1}{a}, \frac{1}{a}\right)$$

