Math 274 In-Class **Worksheet 2b**
Section: 2.3

1. A rock contains two radioactive isotopes A and B, that belong to the same radioactive series; that is A decays into B, which then decays into stable atoms. Assume that the rate at which A decays into B is $40e^{-10t}$ kg/sec. Let $y(t)$ be the mass of B at time t. The rate of decay of B is proportional to the total mass of B present, i.e. $y' = -ky$.

   (a) Write a differential equation modeling the mass of B present at time t. Note, the amount of B is increasing as A decays and creates more B, but simultaneously decreasing as B decays. Assume the constant of proportionality in the decay of B is $k = 2$/sec.

   (b) Express your equation in standard linear form.

   (c) Compute the integrating factor.

   (d) Find a general solution.

   (e) If the mass of B is initially 20 kg, find the mass $y(t)$ of B as a function of t for $t \geq 0$.

2. Consider the first order linear initial value problem

$$y' + \frac{y}{x-1} = \frac{5}{x^2-1}, \qquad y(0) = 1.$$

(a) Find an explicit solution to the initial value problem .

(b) On what interval is your solution unique?

3. For the following initial value problems, are the given solutions unique.

(a) The initial value problem $y' = -\sqrt{y}$, $y(1) = 0$ has solution $y(t) = \frac{1}{4}(x-1)^2$.

(b) The initial value problem $y' + 2xy = x$, $y(0) = 1$ has solution $y(x) = \frac{1}{2}(e^{-x^2} + 1)$.