

1. Find general solutions for the following.

(a)  $y'' - y' - 6y = 0$

$$r^2 - r - 6 = 0$$

$$(r - 3)(r + 2) = 0$$

$$y = C_1 e^{3t} + C_2 e^{-2t}$$

(b)  $y'' - 4y' - 5y = 0$

$$r^2 - 4r - 5 = 0$$

$$(r - 5)(r + 1) = 0$$

$$y = C_1 e^{5t} + C_2 e^{-t}$$

(c)  $y'' - 12y' + 36y = 0$

$$r^2 - 12r + 36 = (r - 6)^2 = 0$$

$$y = C_1 e^{6t} + C_2 t e^{6t}$$

(d)  $2x'' - 3x' - 2x = 0$

$$2r^2 - 3r - 2 = (2r + 1)(r - 2) = 0$$

$$x = C_1 e^{-t/2} + C_2 e^{2t}$$

(e)  $4x'' - x = 0$

$$4r^2 - 1 = (2r - 1)(2r + 1) = 0$$

$$x = C_1 e^{t/2} + C_2 e^{-t/2}$$

(k)  $y''' - 3y'' + 2y' = 0$

$$r^3 - 3r^2 + 2r = 0$$

$$r(r^2 - 3r + 2) = 0$$

$$r(r - 2)(r - 1) = 0$$

$$y = C_1 + C_2 e^t + C_3 e^{2t}$$

(l)  $x^{(iv)} - 5x'' + 4x = 0$

$$r^4 - 5r^2 + 4 = 0$$

$$(r^2 - 4)(r^2 - 1) = 0$$

$$(r - 2)(r + 2)(r - 1)(r + 1) = 0$$

$$x = C_1 e^{2t} + C_2 e^{-2t} + C_3 e^t + C_4 e^{-t}$$

(f)  $x'' = 0$

$$r^2 = 0, r = 0 \text{ repeated}$$

$$x = C_1 + C_2 t$$

(g)  $2y'' - y' = 0$

$$2r^2 - r = r(2r - 1) = 0$$

$$y = C_1 + C_2 e^{2t}$$

(h)  $y' - 2y = 0$

$$r - 2 = 0$$

$$y = C_1 e^{2t}$$

(i)  $4x'' + 12x' + 9x = 0$

$$4r^2 + 12r + 9 = (2r + 3)^2 = 0 \quad r = -3/2 \text{ repeated}$$

$$x = C_1 e^{-3t/2} + C_2 t e^{-3t/2}$$

(j)  $4x'' + 4x' + x = 0$

$$4r^2 + 4r + 1 = (2r + 1)^2 = 0$$

$$x = C_1 e^{-t/2} + C_2 t e^{-t/2}$$

2. Solve the initial value problem .

(a)  $y'' - y' - 2y = 0, \quad y(0) = 5, y'(0) = 4$

$$r^2 - r - 2 = 0$$

$$(r-2)(r+1) = 0$$

$$y = C_1 e^{2t} + C_2 e^{-t}$$

$$y(0) = C_1 + C_2 = 5$$

$$y' = 2C_1 e^{2t} - C_2 e^{-t}$$

$$y'(0) = 2C_1 - C_2 = 4$$

$$3C_1 = 9$$

$$\text{so } C_1 = 3, C_2 = 2$$

$$y = 3e^{2t} - 2e^{-t}$$

(b)  $x'' - 4x' + 4x = 0, \quad \underbrace{x(0) = x'(0) = 0}$

$$r^2 - 4r + 4 = 0$$

$$x \equiv 0, \text{ i.e. } C_1 = C_2 = 0$$

3. Find general solutions for the following.

(a)  $y'' + 4y = 0$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

$$y = C_1 \cos 2t + C_2 \sin 2t$$

(b)  $y'' + 4y' + 5y = 0$

$$r^2 + 4r + 5 = 0$$

$$(r+2)^2 = -1$$

$$r = -2 \pm i$$

$$y = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t$$

(c)  $x'' - 6x' + 13x = 0$

$$r^2 - 6r + 13 = 0$$

$$(r-3)^2 = -4$$

$$r = 3 \pm 2i$$

$$x = C_1 e^{3t} \cos 2t + C_2 e^{3t} \sin 2t$$

4. Solve the initial value problem .

(a)  $y'' + 2y' + 2y = 0, \quad y(0) = 2, y'(0) = 0$

$$r^2 + 2r + 2 = 0$$

$$(r+1)^2 = -1$$

$$r = -1 \pm i$$

$$y = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t$$

$$y(0) = 2 \Rightarrow C_1 = 2$$

$$y' = -2e^{-t} \cos t - 2e^{-t} \sin t + C_2 e^{-t} \cos t - C_2 e^{-t} \sin t$$

$$y'(0) = 0 \Rightarrow C_2 = 2$$

$$y = 2e^{-t} (\cos t + \sin t)$$