

## Method of Undetermined Coefficients

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{rt}$$

where  $P_m(t)$  is a polynomial of degree  $m$ , use the form

$$y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{rt};$$

if  $r$  is not a root of the associated auxiliary equation, take  $s = 0$ ; if  $r$  is a simple root, take  $s = 1$ ; and if  $r$  is a double root, take  $s = 2$ .

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{\alpha t} \cos \beta t + Q_n(t)e^{\alpha t} \sin \beta t$$

where  $P_m(t)$  and  $Q_n(t)$  are polynomials of degree  $m$  and  $n$ , respectively, use the form

$$y_p(t) = t^s \left( A_k t^k + \cdots + A_1 t + A_0 \right) e^{\alpha t} \cos \beta t + t^s \left( B_k t^k + \cdots + B_1 t + B_0 \right) e^{\alpha t} \sin \beta t;$$

where  $k$  is the larger of  $m$  and  $n$ . If  $\alpha + i\beta$  is not a root of the associated auxiliary equation, take  $s = 0$ ; if so take  $s = 1$ .

1. Find the appropriate form using the Method of Undetermined Coefficients for a particular solution to the following. **Do not** solve for the unknown constants.

(a)  $y'' - 4y' + 4y = t \sin 3t$

(b)  $y'' - 4y' + 4y = te^t + 3e^{4t}$

(c)  $y'' - y' - 6y = e^{3t} + 7t^2$

(d)  $y'' + 9 = \sin 3t$

(e)  $y'' + 9 = e^{2t} \cos 3t$

**STOP!** Don't move on until you know these are correct.

2. Find a general solution for the following.

(a)  $y'' + y = 4$

(b)  $y'' + y = 4t$

(c)  $y'' = 4$

(d)  $y'' + 6y' + 8y = 16 + 6e^{-2t}$

(e)  $y'' + 6y' + 8y = \cos 2t$