Math 274 In-Class **Worksheet 6a**
Sections: 4.6

Variation of Parameters

If y_1 and y_2 are linearly independent solutions to $y'' + p(t)y' + q(t)y = 0$, then a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is given by

$$y_p(t) = y_1(t) \int \frac{-g(t)y_2(t)}{W[y_1, y_2](t)}\, dt + y_2(t) \int \frac{g(t)y_1(t)}{W[y_1, y_2](t)}\, dt.$$

1. We are interested in solving the initial value problem

$$y'' + 9y = \csc 3t, \qquad y(\pi/6) = y'(\pi/6) = 0. \tag{1}$$

   (a) Find a general solution to the associated homogeneous equation, i.e $y'' + 9y = 0$.

   (b) Find a particular solution to $y'' + 9y = \csc 3t$.

   (c) Find a general solution to $y'' + 9y = \csc 3t$.

   (d) Find the solution to the initial value problem (1).

2. Consider the equation

$$y'' + 9y = 9 \sin 3t.$$

(a) Find a particular solution using Variation of Parameters. You may find the following trigonometric identities useful.

- $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$
- $\sin 2x = 2 \sin x \cos x$

(b) Find a particular solution using the Method of Undetermined Coefficients.