

Sections: 4.9, and more

1. Consider mass-spring systems characterized by the following initial value problems. Characterize each as undamped, underdamped, critically damped, or overdamped. Solve each initial value problem and sketch the solution.

(a)  $x'' + 2x' + x = 0$ ,  $x(0) = x'(0) = 1$

(b)  $x'' + 3x' + 2x = 0$ ,  $x(0) = x'(0) = 1$

(c)  $x'' + 4x' + 5x = 0$ ,  $x(0) = x'(0) = 1$

(d)  $x'' + 4x = 0$ ,  $x(0) = x'(0) = 1$

2. Consider a mass-spring system that has an external forcing function that turns off at  $t = \pi/2$  modeled by the following initial value problem

$$y'' + 2y' + 2y = \begin{cases} 1, & t < \pi/2 \\ 0, & \pi/2 < t \end{cases}, \quad y(0) = y'(0) = 0.$$

Find a solution to the initial value problem. Note, your solution will be piecewise defined.