

Sections: 7.2,7.3

1. Use the definition to determine the Laplace transform of $f(t) = \begin{cases} t, & 0 \leq t < 4 \\ e^{-t}, & 4 < t \end{cases}$

2. 2 Use the provided table and linearity to determine the Laplace transform of the following functions.

(a) $f(t) = 4t^2 - 7e^{2t} \cos 3t$

(b) $g(t) = 6 + t^8 e^{-3t}$

(c) $h(t) = 5te^{2t} \sin 3t$

(d) $j(t) = \cos^2 bt$

[HINT: $\cos^2 x = (1 + \cos 2x)/2$.]

3. For each of the following choose all that apply; the function is not piecewise continuous on $[0, \infty)$ (**NPC**), the function is not of exponential order (**NEO**), and/or the function has a Laplace transform (**LT**).

(a) **NPC** / **NEO** / **LT** : $f(t) = \tan t$

(b) **NPC** / **NEO** / **LT** : $f(t) = e^{\sin t}$

(c) **NPC** / **NEO** / **LT** : $f(t) = 14e^{t^2+7}$

(d) **NPC** / **NEO** / **LT** : $f(t) = \begin{cases} 4e^{3t}, & 0 < t < 2 \\ 12e^t, & 2 < t \end{cases}$

4. The inverse Laplace transform is defined as you would expect it to be¹. For example, since $\mathcal{L}\{\sin 3t\} = \frac{3}{s^2 + 9}$, we have $\mathcal{L}^{-1}\left\{\frac{3}{s^2 + 9}\right\} = \sin 3t$. Find the inverse Laplace transform of the following.

(a) $F(s) = \frac{s}{s^2 + 4}$

(b) $G(s) = \frac{8s}{s^2 + 4}$

(c) $H(s) = \frac{6}{s^4}$

(d) $K(s) = \frac{1}{s^4}$

(e) $W(s) = \frac{6 - 7s}{s^2 + 9}$

[HINT: separate the fraction.]

¹There are some technical complications we will address next week, but for now we will ignore them and assume it works as we expect.