Math 274 In-Class **Worksheet 8p**

Section: 7.5

1. Apply the Laplace transform to the initial value problem

$$y'' + 3y' = 7, \qquad y(0) = 1, y'(0) = -3$$

to express $Y(s) = \mathscr{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; for example, (1) below is of this form. **Do not find the inverse Laplace transform.**

$$s^2 Y - s + 3 + 3sY - 3 = \frac{7}{s}$$

$$Y(s^2 + 3s) = \cancel{\frac{7+s^2}{s}} \quad \frac{7+s^2}{s}$$

$$Y = \frac{7+s^2}{s^2(s+3)}$$

2. Applying the Laplace transform to the initial value problem

$$y'' - 6y' + 9y = e^{2t}, \qquad y(0) = 3, y'(0) = 4$$

gives the following

$$Y(s) = \frac{3s^2 - 20s + 29}{(s-2)(s^2 - 6s + 9)}. \tag{1}$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{3s^2 - 20s + 29}{(s-2)(s-3)^2} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{(s-3)^2}$$

$$3s^2 - 20s + 29 = A(s-3)^2 + B(s-2)(s-3) + C(s-2)$$

$s=2$: $12 - 40 + 29 = A$ so $A = 1$

$s=3$: $27 - 60 + 29 = C$ so $C = -4$

Eq. Coeff

s^2 : $3 = A + B$ so $B = 2$

$$y = e^{2t} + 2e^{3t} - 4te^{3t}$$

3. Use the method of Laplace Transforms to solve the following initial value problems.

(a) $y'' + 4y = 4t^2 - 4t + 10, \qquad y(0) = 0, y'(0) = 3$

$$s^2 Y - 3 + 4Y = \frac{8}{s^3} - \frac{4}{s^2} + \frac{10}{s}$$

$$Y = \frac{8 - 4s + 10s^2 + 3s^3}{s^3(s^2+4)} = \frac{2}{s^3} - \frac{1}{s^2} + \frac{2}{s} - \frac{2s}{s^2+2^2} + \frac{4}{s^2+2^2}$$ ← Use a computer!

$$y = t^2 - t + 2 - 2\cos 2t + 2\sin 2t$$

(b) $y'' - 4y' + 5y = 4e^{3t}, \qquad y(0) = 2, y'(0) = 7$

$$s^2 Y - 2s - 7 - 4(sY - 2) + 5Y = \frac{4}{s-3}$$

$$Y(s^2 - 4s + 5) = \frac{4}{s-3} + 2s - 1 = \frac{4}{s-3} + \frac{(2s-1)(s-3)}{s-3} = \frac{2s^2 - 7s + 7}{s-3}$$

$$Y = \frac{2s^2 - 7s + 7}{(s-3)(s^2 - 4s + 5)} = \frac{A}{s-3} + \frac{Bs + C}{s^2 - 4s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 4s + 5) + (Bs + C)(s - 3)$$

$s = 3$: $18 - 21 + 7 = A(2)$ so $A = 2$

s^2 : $2 = A + B$ so $B = 0$

s^0 : $7 = 5A - 3C$ so $C = 1$

$$Y = \frac{2}{s-3} + \frac{1}{(s-2)^2 + 1^2} \quad \text{so} \quad y = 2e^{3t} + e^{2t}\sin t$$