1. Express the following functions using step functions and determine their Laplace transforms.

   (a) $f(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 < t < 2 \\ 2, & 2 < t \end{cases}$

   (b) $g(t) = \begin{cases} t, & t < \pi \\ e^{3t}\cos 2t, & \pi < t \end{cases}$

2. Determine the inverse Laplace transform for the following.

   (a) $H(s) = \dfrac{1}{s} + \dfrac{e^{-2s}}{s^2} + \dfrac{e^{-4s}}{s^3}$

   (b) $K(s) = \dfrac{8e^{4-2s}}{(s-2)(s^2+4)}$

3. Applying the Laplace transform to the initial value problem

$$y'' + 4y = f(t), \qquad y(0) = 0, y'(0) = 0$$

with

$$f(t) = \begin{cases} 10e^t, & t < 1 \\ -8e^{2t}, & 1 < 2 < t \\ 0, & 2 < t \end{cases}$$

gives

$$Y(s) = \frac{10}{(s-1)(s^2+4)} + e^{-s}\left[\frac{-8e^2}{(s-2)(s^2+4)} - \frac{10e}{(s-1)(s^2+4)}\right] + e^{-2s}\left[\frac{8e^4}{(s-2)(s^2+4)}\right].$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

You should reference the partial fraction decomposition in 2(b). Additionally, a routine exercise shows $\frac{10}{(s-1)(s^2+4)} = \frac{2}{s-1} - \frac{2s+2}{s^2+4}$.

Exercise: Using your favorite piece of technology, generate a picture of the graph of y. It should look something like this.