Math 274 In-Class **Worksheet ~~13~~ 12**

Section: 9.1, 9.2, 9.3

1. (a) Write the following second order equation as a system of first order equations: $y''+3y'-y=0$.

$x_1 = y$

$x_2 = y'$

$x_1' = x_2$

$x_2' = x_1 - 3x_2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b) Write the following second order equation as a system of first order equations: $y''+3y=e^t$.

$x_1 = y$

$x_2 = y'$

$x_1' = x_2$

$x_2' = -3x_1 + e^t$

$$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -3 & 0 \end{bmatrix}\vec{x} + \begin{bmatrix} 0 \\ e^t \end{bmatrix}$$

(c) Write the following third order equation as a system of first order equations: $y'''+2y''-2y=0$.

$x_1 = y$

$x_2 = y'$

$x_3 = y''$

$$\vec{x}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ +2 & 0 & -2 \end{bmatrix}\vec{x}$$

(d) Write the coupled system in matrix notation

$$x''+3x-y=0$$
$$y''+2y-2x=0$$

$x_1 = x$

$x_2 = x'$

$x_3 = y$

$x_4 = y'$

$$\vec{x}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -2 & 0 \end{bmatrix}\vec{x}$$

2. Let M be the 3×3 matrix

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Compute the determinant of M.

$$\det M = |M| = 0 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= 1 - 2 = -1$$

(b) Determine the inverse of M. Verify $M^{-1}M = I$.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right] \begin{array}{l} R_3 - R_1 \mapsto R_1 \\ R_2 - R_1 \mapsto R_2 \\ 2R_1 - R_2 \mapsto R_3 \end{array}$$

(c) Solve the system of linear equations:

$$\begin{aligned} b + c &= 5 \\ 2b + c &= 6 \\ a + \ b + c &= 7 \end{aligned}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

3. Find the inverse of the 4×4 matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 3 & 3 & 3 \end{bmatrix}$$

$$\det A = 0 - 1 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{vmatrix} - 0 = 0$$

, so A is singular, i.e., does not have an inverse.

4. (a) Find the equation of the line passing through the points $(2, 4)$ and $(5, 13)$.

$$y - 4 = 3(x - 2)$$

– or –

$$y = 3x - 2$$

(b) For $t > 0$, write the following function in terms of step functions

$$f(t) = \begin{cases} t, & t < 2 \\ 1/2, & 2 < t < 3 \\ e^{t-2}, & 3 < t \end{cases}$$

$$f(t) = t + u(t-2)\left(t - \tfrac{1}{2}\right) + u(t-3)\left(e^{t-2} - \tfrac{1}{2}\right)$$

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(c) Compute $\mathscr{L}\{f(t)\}$, for $f(t)$ as above.

$$F(s) = \frac{1}{s^2} + e^{-2s}\mathcal{L}\left\{(t+2) - \tfrac{1}{2}\right\} + e^{-3s}\mathcal{L}\left\{e^{(t+3)-2} - \tfrac{1}{2}\right\}$$

$$= \frac{1}{s^2} + e^{-2s}\left[\frac{1}{s^2} + \frac{3}{2s}\right] + e^{-3s}\left[\frac{e}{s-1} - \frac{1}{2s}\right]$$

(d) For $t > 0$, let $g(t)$ be the periodic function of period 2 given on a fundamental period by

$$g(t) = 9 - e^t, \ 0 < t < 2.$$

Compute $\mathscr{L}\{g(t)\}$.

$$g_2 = 9 - e^t - u(t-2)(9 - e^t)$$

$$G_2 = \frac{9}{s} - \frac{1}{s+1} - e^{-2s}\left(\frac{9}{s} - \frac{e^2}{s+1}\right)$$

$$G = \frac{8s + 9 - e^{-2s}(9s + 9 - e^2 s)}{s(s+1)(1 - e^{-2s})}$$