Math 274 In-Class **Worksheet 8a**

Sections: 7.4

1. Determine the inverse Laplace transform of the following.

(a) $F(s) = \dfrac{4}{s+3}$

$f(t) = 4e^{-3t}$

(b) $G(s) = \dfrac{3}{2s+1} = \dfrac{3}{2} \cdot \dfrac{1}{s+1/2}$

$g(t) = \dfrac{3}{2} e^{-t/2}$

(c) $H(s) = \dfrac{2}{3-s} = \dfrac{-2}{s-3}$

$h(t) = -2e^{3t}$

(d) $J(s) = \dfrac{2}{(s-3)^4} = \dfrac{2}{3!} \cdot \dfrac{3!}{(s-3)^4}$

$j(t) = \dfrac{1}{3} t^3 e^{3t}$

(e) $K(s) = \dfrac{2}{s^2+3} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{s^2+(\sqrt{3})^2}$

$k(t) = \dfrac{2}{\sqrt{3}} \sin(\sqrt{3}t)$

(f) $M(s) = \dfrac{2s}{s^2+3}$

$m(t) = 2\cos(\sqrt{3}t)$

(g) $N(s) = \dfrac{2s+2}{s^2+3}$

$n(t) = m(t) + k(t)$ (see above)

(h) $P(s) = \dfrac{6s}{s^2+4s+6} = \dfrac{6(s+2) - 12}{(s+2)^2 + (\sqrt{2})^2}$

$p(t) = 6e^{-2t} \cos\sqrt{2}t - \dfrac{12}{\sqrt{2}} e^{-2t} \sin\sqrt{2}t$

(i) $Q(s) = \dfrac{4s+2}{s^3+2s^2} = \dfrac{3/2}{s} + \dfrac{1}{s^2} - \dfrac{3/2}{s+2}$

$q(t) = \dfrac{3}{2} + t - \dfrac{3}{2} e^{-2t}$

2. Consider the initial value problem

$$y'' + 4y = 8t - 4, \qquad y(0) = 1, y'(0) = 0. \tag{1}$$

(a) Applying the Laplace transform to the initial value problem (1) gives the following

$$\left[s^2Y(s) - s\right] + 4Y(s) = \frac{8}{s^2} - \frac{4}{s}. \tag{2}$$

Solve equation (2) above for $Y(s)$ and then determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$ which is the solution to the initial value problem.

$Y(s^2+4) = \dfrac{8-4s+s^3}{s^2}$ so $Y = \dfrac{8-4s+s^3}{s^2(s^2+4)}$

$\dfrac{8-4s+s^3}{s^2(s^2+4)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{Cs+D}{s^2+4}$

$8-4s+s^3 = As(s^2+4) + B(s^2+4) + (Cs+D)s^2$

Let $s=0$: $8 = 4B$ so $B=2$

Let $s=2i$: $8-8i-8i = -8Ci - 4D$ so $C=2,\ D=-2$

Eq. Coeff s^3 : $1 = A + C$ so $A=-1$

$Y = \dfrac{-1}{s} + \dfrac{2}{s^2} + \dfrac{2s}{s^2+4} - \dfrac{2}{s^2+4}$ so $y = -1 + 2t + 2\cos 2t - \sin 2t$

(b) Use methods from Chapter 4 to solve the the initial value problem (1).

$r^2+4=0$

so

$y = C_1 \cos 2t + C_2 \sin 2t$

$y_p = At+B$

$y_p' = A$

$y_p'' = 0$

Substituting gives

$4At + 4B = 8t - 4$

so $A=2,\ B=-1$

$y = C_1 \cos 2t + C_2 \sin 2t + 2t - 1$

$y(0)=1 \Rightarrow C_1 = 2$

$y' = -4\sin 2t + 2C_2 \cos 2t + 2$

$y'(0)=0 \Rightarrow C_1 = -1$

so

$y = 2\cos 2t - \sin 2t + 2t - 1$