Math 274 In-Class **Worksheet 13**

Section: 9.4

1. (a) Show that the following vectors are linearly dependent

$$\left\{ \begin{bmatrix} 1 \\ 4 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 7 \\ 10 \\ -4 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 5 \\ -4 \end{bmatrix} \right\}$$

   (b) Are the following vectors linearly independent?

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix} \right\}$$

   (c) Are the following vectors linearly independent on $(-\infty, \infty)$?

$$\left\{ \begin{bmatrix} te^{-t} \\ e^{-t} \end{bmatrix}, \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} \right\}$$

2. On which interval does the following initial value problem have a unique solution?

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} \tan t & 0 \\ t & \ln t \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e^t \\ 1/t \end{bmatrix}, \quad \begin{bmatrix} x_1(\pi/4) \\ x_2(\pi/4) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3. (a) Let $\mathbf{X}(t)$ be a fundamental matrix for $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Show that $\mathbf{x}(t) = \mathbf{X}(t)\mathbf{X}^{-1}(t_0)\mathbf{x}_0$ is a solution to the ivp given by the initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$.

   (b) Verify $\mathbf{X}(t)$ is a fundamental matrix and solve the ivp.

$$\mathbf{x}' = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad \mathbf{X}(t) = \begin{bmatrix} e^{-t} & e^{5t} \\ -e^{-t} & e^{5t} \end{bmatrix}$$