

Section: 9.1, 9.2, 9.3

1. (a) Write the following second order equation as a system of first order equations:  $y'' + 3y' - y = 0$ .

(b) Write the following second order equation as a system of first order equations:  $y'' + 3y = e^t$ .

(c) Write the following third order equation as a system of first order equations:  $y''' + 2y'' - 2y = 0$ .

(d) Write the coupled system in matrix notation

$$x'' + 3x - y = 0$$

$$y'' + 2y - 2x = 0$$

2. Let  $M$  be the  $3 \times 3$  matrix

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) Compute the determinant of  $M$ .

(b) Determine the inverse of  $M$ . Verify  $M^{-1}M = I$ .

(c) Solve the system of linear equations:

$$\begin{aligned} b + c &= 5 \\ 2b + c &= 6 \\ a + b + c &= 7 \end{aligned}$$

3. Find the inverse of the  $4 \times 4$  matrix

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 3 & 3 & 3 \end{bmatrix}$$

4. (a) Find the equation of the line passing through the points  $(2, 4)$  and  $(5, 13)$ .

(b) For  $t > 0$ , write the following function in terms of step functions

$$f(t) = \begin{cases} t, & t < 2 \\ 1/2, & 2 < t < 3 \\ e^{t-2}, & 3 < t \end{cases}$$

(c) Compute  $\mathcal{L}\{f(t)\}$ , for  $f(t)$  as above.

(d) For  $t > 0$ , let  $g(t)$  be the periodic function of period 2 given on a fundamental period by

$$g(t) = 9 - e^t, \quad 0 < t < 2.$$

Compute  $\mathcal{L}\{g(t)\}$ .