1. Integrate.

(a) \[5\int y\sqrt{y + 3}\,dy = \int (u - 3)^{3/2} \, du = \int \left(\frac{2}{3} u^{3/2} - \frac{2}{5} u^{5/2}\right) \, du\]

Let \( u = y + 3 \)

so \( du = dy \)

\[\frac{y}{3} = u - 3\]

\[= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \, + C\]

(b) \[6\int_0^{\pi/2} \sin (\pi - 2x) \, dx = -\frac{1}{2} \int_0^{\pi} \sin u \, du\]

Let \( u = \pi - 2x \)

so \( du = -2 \, dx \)

\[\sin \frac{u}{2} \rightarrow \frac{u}{\pi}\]

\[= \frac{1}{2} \cos u \int_0^{\pi} = \frac{1}{2} \left( \cos 0 - \cos \pi \right)\]

\[= \frac{1}{2} (1 - (-1)) = 1\]

(c) \[6\int \frac{6x^5}{1 + x^6} \, dx = \int \frac{du}{u} = \ln |u| + C\]

Let \( u = 1 + x^6 \)

so \( du = 6x^5 \, dx\)

\[= \ln |1 + x^6| + C\]

(d) \[6\int \frac{3x^2}{1 + x^6} \, dx = \int \frac{du}{1 + u^2} = \arctan u + C\]

Let \( u = x^3 \)

so \( du = 3x^2 \, dx\)

\[= \arctan x^3 + \ln |1 + x^6| + C\]

(e) \[2\int \frac{3x^2 + 6x^5}{1 + x^6} \, dx \text{ [See above.]}\]

\[= \arctan x^3 + \ln |1 + x^6| + C\]
2. [10] Sketch the region enclosed by \( y = 5x - x^2 \) and \( y = x^2 - 3x \) and compute its area.

\[
-Area = \int_{0}^{4} \left( (5x - x^2) - (x^2 - 3x) \right) \, dx
\]

\[
= \int_{0}^{4} \left[ 8x - 2x^2 \right] \, dx
\]

\[
= 4x^2 - \frac{2}{3} x^3 \bigg|_{0}^{4}
\]

\[
= 64 - \frac{2}{3} (64) = \frac{64}{3}
\]

**Intersections**

\( 5x - x^2 = x^2 - 3x \)

\( 8x = 2x^2 \)

\( x = 0 \) or \( x = 4 \)

3. [10] Show the volume of a sphere of radius \( R \) is \( V = \frac{4}{3} \pi R^3 \) by computing the volume of the solid generated by rotating the semicircle \( y = \sqrt{R^2 - x^2} \) about the \( x \)-axis.

\[
V = \int_{-R}^{R} \pi \left( R^2 - x^2 \right) \, dx
\]

\[
= \pi \left( R^3 x - \frac{x^3}{3} \right) \bigg|_{-R}^{R}
\]

\[
= \pi \left[ (R^3 - \frac{R^3}{3}) - (-R^3 + \frac{R^3}{3}) \right]
\]

\[
= \frac{4}{3} \pi R^3
\]
4. [10] Find the volume of the wedge in the figure by integrating the area of vertical cross sections.

By similar \( \Delta \)'s we have

\[
\frac{h}{4-x} = \frac{2}{4} \quad \text{so} \quad h = \frac{1}{2} (4-x)
\]

and

\[
\frac{b}{4-x} = \frac{3}{4} \quad \text{so} \quad b = \frac{3}{4} (4-x)
\]

so the area of \( \text{is} \quad \frac{1}{2} \left( \frac{3}{4} (4-x) \right) \left( \frac{1}{2} (4-x) \right) \)

\[
= \frac{3}{16} (4-x)^2
\]

The volume is then given by

\[
V = \int_0^4 \frac{3}{16} (4-x)^2 \, dx = \frac{3}{16} \left[ \frac{1}{4} x^4 - 4x^3 + 6x^2 \right]_0^4 = \frac{3}{4}
\]

5. [10] The region bounded by the graphs of \( y = x^2 + 1 \), \( x = 1 \), the \( x \)-axis, and the \( y \)-axis is revolved around the \( y \)-axis, see figure. Compute the volume of the resulting solid.

Using shells

\[
V = \int_0^1 2\pi x (x^2 + 1) \, dx = 2\pi \left[ \frac{1}{4} x^4 + \frac{1}{2} x^2 \right]_0^1 = 2\pi \left( \frac{1}{4} + \frac{1}{2} \right)
\]

\[
= 2\pi \left( \frac{3}{4} \right) = \frac{3\pi}{2}
\]
6. Consider the region bounded by the curves \( y = x^2 \) and \( y = \sqrt{x} \).

(a) [5] Sketch the region.

(b) [5] Express the volume of the solid obtained by rotating the region about the \( x \)-axis using the Disk Method. **Do not integrate.**

\[
\int_0^1 \pi \left[ (x^2)^2 - (x^2) \right] \, dx
\]

(c) [5] Express the volume of the solid obtained by rotating the region about the \( x \)-axis using the Shell Method. **Do not integrate.**

\[
\int_0^1 2\pi (y) (\sqrt{y} - y^2) \, dy
\]

(d) [5] Express the volume of the solid obtained by rotating the region about the line \( x = 2 \) using the Disk Method. **Do not integrate.**

\[
\int_0^1 \pi \left[ (2 - y^2)^2 - (2 - \sqrt{y})^2 \right] \, dy
\]

(e) [5] Express the volume of the solid obtained by rotating the region about the line \( x = 2 \) using the Shell Method. **Do not integrate.**

\[
\int_0^1 2\pi (2 - x) (\sqrt{x} - x^2) \, dx
\]