Math 172 | Exam 1 | Name: ______________

16 May 2013 | Show Appropriate Work | Point Values in [Brackets]

1. Integrate.

(a) [5] $\int y\sqrt{y+3}\,dy$ $= \int (u-3)u^{1/2}\,du = \int (u^{3/2} - 3u^{1/2})\,du$

Let $u = y+3$

so $du = dy$

& $y = u-3$

$= \frac{2}{5}u^{5/2} - 3\left(\frac{2}{3}u^{3/2}\right) + C$

$= \frac{2}{5}(y+3)^{5/2} - 2(y+3)^{3/2} + C$

(b) [6] $\int_0^{\frac{\pi}{2}} \sin(\pi - 2x)\,dx$ $= -\frac{1}{2}\int_{\pi}^{0} \sin u\,du$

Let $u = \pi - 2x$

so $du = -2\,dx$

$$\begin{array}{c|c} x & u \\ \hline 0 & \pi \\ \pi/2 & 0 \end{array}$$

$= \frac{1}{2}\cos u\Big|_{\pi}^{0} = \frac{1}{2}(\cos 0 - \cos\pi)$

$= \frac{1}{2}(1-(-1)) = 1$

(c) [6] $\int \frac{6x^5}{1+x^6}dx$ $= \int \frac{du}{u} = \ln|u| + C$

Let $u = 1+x^6$

so $du = 6x^5\,dx$

$= \ln(1+x^6) + C$

(d) [6] $\int \frac{3x^2}{1+x^6}dx$ $= \int \frac{du}{1+u^2} = \arctan u + C$

Let $u = x^3$

so $du = 3x^2\,dx$

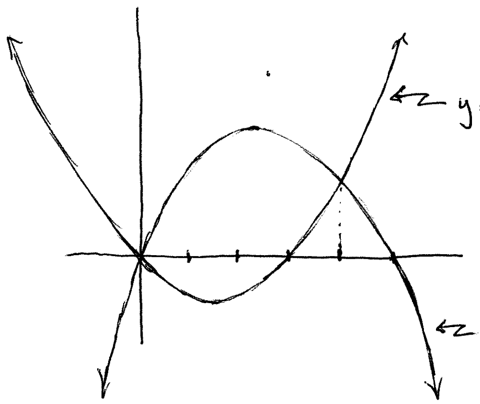
$= \arctan x^3 + C$

(e) [2] $\int \frac{3x^2+6x^5}{1+x^6}dx$ [See above.]

$= \arctan x^3 + \ln|1+x^6| + C$

2. [10] Sketch the region enclosed by $y = 5x - x^2$ and $y = x^2 - 3x$ and compute its area.

$= x(5-x) \qquad = x(x-3)$



$$\text{Area} = \int_0^4 \left((5x - x^2) - (x^2 - 3x) \right) dx$$

$$= \int_0^4 [8x - 2x^2] \, dx$$

$$= 4x^2 - \frac{2}{3}x^3 \Big|_0^4$$

$$= 64 - \frac{2}{3}(64) = \frac{64}{3}$$

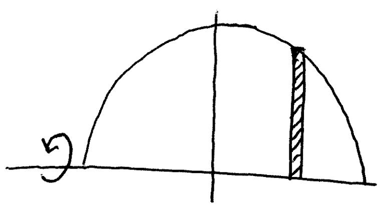
Intersections

$5x - x^2 = x^2 - 3x$

$8x = 2x^2$

$4 = x$ or $x = 0$

3. [10] Show the volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$ by computing the volume of the solid generated by rotating the semicircle $y = \sqrt{R^2 - x^2}$ about the x-axis.



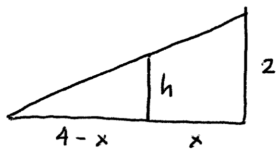
$$V = \int_{-R}^{R} \pi (R^2 - x^2) \, dx$$

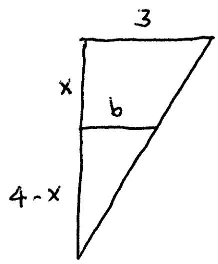
$$= \pi \left(R^2 x - \frac{x^3}{3} \right) \Big|_{-R}^{R}$$

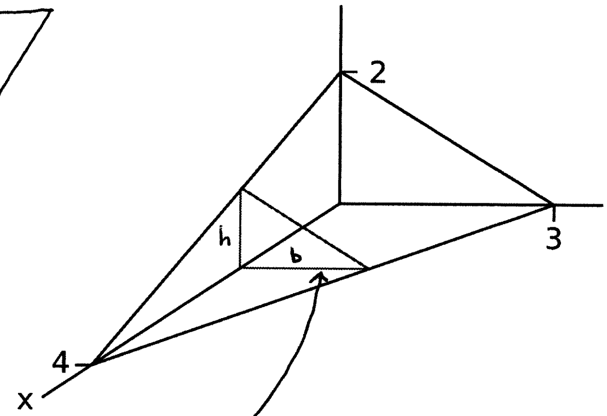
$$= \pi \left[\left(R^3 - \frac{R^3}{3} \right) - \left(-R^3 + \frac{R^3}{3} \right) \right]$$

$$= \frac{4}{3} \pi R^3$$

4. [10] Find the volume of the wedge in the figure by integrating the area of vertical cross sections.







By similar △'s we have

$$\frac{h}{4-x} = \frac{2}{4} \quad \text{so} \quad h = \frac{1}{2}(4-x)$$

and

$$\frac{b}{4-x} = \frac{3}{4} \quad \text{so} \quad b = \frac{3}{4}(4-x)$$

so the area of is $\frac{1}{2}\left(\frac{3}{4}(4-x)\right)\left(\frac{1}{2}(4-x)\right)$

$$= \frac{3}{16}(4-x)^2$$

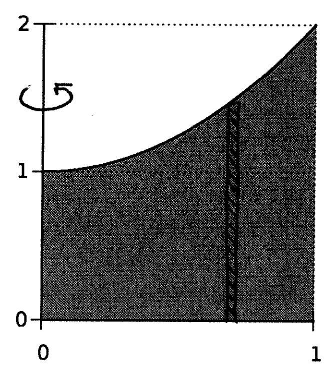
The volume is then given by

$$V = \int_0^4 \frac{3}{16}(4-x)^2\,dx = -\frac{3}{16}\int_4^0 u^2\,du = -\frac{1}{16}u^3\Big|_4^0 = 4$$

$u = 4-x$ $\quad 0 \mapsto 4$

$du = -x\,dx$ $\quad 4 \mapsto 0$

5. [10] The region bounded by the graphs of $y = x^2 + 1$, $x = 1$, the x-axis, and the y-axis is revolved around the y-axis, see figure. Compute the volume of the resulting solid.



Using shells

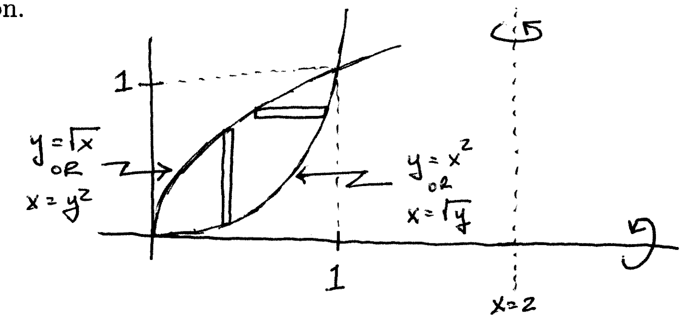
$$V = \int_0^1 2\pi\, x\,(x^2+1)\,dx = 2\pi\int_0^1 (x^3 + x)\,dx$$

$$= 2\pi\left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Bigg|_0^1 = 2\pi\left(\frac{1}{4} + \frac{1}{2}\right)$$

$$= 2\pi\left(\frac{3}{4}\right) = \frac{3\pi}{2}$$

6. Consider the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$.

(a) [5] Sketch the region.



(b) [5] Express the volume of the solid obtained by rotating the region about the x-axis using the Disk Method. **Do not integrate.**

$$\int_0^1 \pi\left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

(c) [5] Express the volume of the solid obtained by rotating the region about the x-axis using the Shell Method. **Do not integrate.**

$$\int_0^1 2\pi (y)(\sqrt{y} - y^2)\, dy$$

(d) [5] Express the volume of the solid obtained by rotating the region about the line $x = 2$ using the Disk Method. **Do not integrate.**

$$\int_0^1 \pi\left[(2-y^2)^2 - (2-\sqrt{y})^2\right] dy$$

(e) [5] Express the volume of the solid obtained by rotating the region about the line $x = 2$ using the Shell Method. **Do not integrate.**

$$\int_0^1 2\pi (2-x)(\sqrt{x} - x^2)\, dx$$