Math 172 | Exam 2 | Name: ________________
23 May 2013 | Show Appropriate Work | Point Values in [Brackets]

1. Integrate.

§7.1 #19

(a) [6] $\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

§7.1 #25

(b) [6] $\int x \sec^2 x \, dx = x\tan x - \int \tan x \, dx = x \tan x - \ln|\sec x| + C$

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \tan x$

– or –

$= x\tan x + \ln|\cos x| + C$

(c) [8] $\int_0^{\frac{\pi}{2}} x \cos x \, dx = x \sin x \Big|_0^{\frac{\pi}{2}} - \int_0^{\pi/2} \sin x \, dx$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$= \frac{\pi}{2} + \cos x \Big|_0^{\pi/2} = \frac{\pi}{2} - 1$

2. Integrate.

§7.2

#17

(a) [6] $\int \tan^2 x \sec^4 x \, dx$ $= \int \tan^2 x \,(1+\tan^2 x)\sec^2 x \, dx$

let $u = \tan x$

so $du = \sec^2 x \, dx$

$= \int (u^2 + u^4)\, du = \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$

§7.2

#5

(b) [6] $\int \sin^3 t \cos^3 t \, dt$

$= \int \sin^3 t \,(1-\sin^2 t)\cos t \, dt$

$u = \sin t$ so $du = \cos t \, dt$

$= \int u^3 (1-u^2)\, du = \int (u^3 - u^5)\, du$

$= \frac{1}{4}\sin^4 t - \frac{1}{6}\sin^6 t + C$

or

$= \int \cos^3 t \,(1-\cos^2 t)\sin t \, dt$

$u = \cos t$ so $du = -\sin t \, dt$

$= -\int u^3 (1-u^2)\, du = \int (u^5 - u^3)\, du$

$= \frac{1}{6}\cos^6 t - \frac{1}{4}\cos^4 t + C$

§7.2

#43

(c) [8] $\int t \sin^3 (t^2)\, dt$ $= \frac{1}{2}\int \sin^3 u \, du = \frac{1}{2}\int (1-\cos^2 u)\sin u \, du$

$u = t^2$

$du = 2t\, dt$

let $x = \cos u$

$dx = -\sin u \, du$

$= \frac{1}{2}\int (x^2 - 1)\, dx = \frac{1}{2}\left(\frac{x^3}{3} - x\right) + C$

$= \frac{\cos^3 t^2}{6} - \frac{\cos t^2}{2} + C$

The better version of §7.3 #39

3. Integrate.

(a) [10] $\int \frac{dx}{\sqrt{x^2+6x}}$ $= \int \frac{dx}{\sqrt{(x+3)^2-3^2}} = \int \frac{3\sec\theta\tan\theta}{3\tan\theta}\,d\theta = \int \sec\theta\,d\theta$

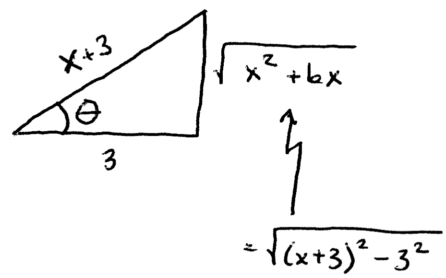
$x^2+6x = x^2+6x+9-9$

$= (x+3)^2-3^2$

Let $x+3 = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$= \ln|\sec\theta + \tan\theta| + C$



$= \ln\left|\frac{x+3}{3} + \frac{\sqrt{x^2+6x}}{3}\right| + C$

$= \ln\left|x+3+\sqrt{x^2+6x}\right| + C$

(b) [10] $\int e^{\sqrt{t}}\,dt$ [Hint: Let $x = \sqrt{t}$]

§7.1 #37

let $x = \sqrt{t}$

so $dx = \frac{1}{2\sqrt{t}}\,dt$

& $2x\,dx = dt$

$= \int 2xe^x\,dx$

$u = 2x$ $dv = e^x\,dx$

$du = 2dx$ $v = e^x$

$= 2xe^x - \int 2e^x\,dx$

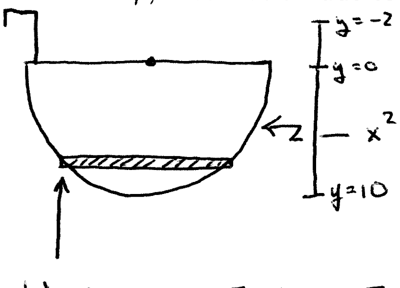
$= 2xe^x - 2e^x + C$

$= 2\sqrt{t}\,e^{\sqrt{t}} - 2e^{\sqrt{t}} + C$

§6.5

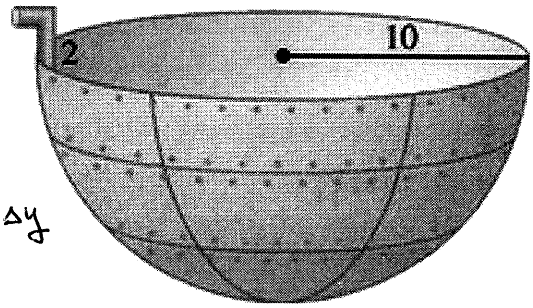
#19

4. [15] Calculate the work (in joules) required to pump all of the water out of the full hemispherical tank in the figure below; water exits through the spout. Distances are in meters, the density of water is ρ, acceleration due to gravity is g. Please specify your coordinate system.



$x^2+y^2=10^2$

so $x=\sqrt{10^2-y^2}$



WORK ON SLICE $= \pi \left(\sqrt{10^2-y^2}\right)^2 \rho g (y+2)\Delta y$

$$\text{Total Work} = \int_0^{10} \pi\rho g (y+2)(100-y^2)\,dy = \pi\rho g \int_0^{10} (200+100y-2y^2-y^3)\,dy$$

$$= \pi\rho g \left[200y + 50y^2 - \frac{2}{3}y^3 - \frac{1}{4}y^4\right]\Bigg|_0^{10} = \pi\rho g\left[2000+5000-\frac{2}{3}(1000)-2500\right]$$

$$= 500\pi\rho g\left[4+10-\frac{4}{3}-5\right] = 500\pi\rho g\left(\frac{23}{3}\right) = \frac{11500\pi\rho g}{3}$$

§7.3

#29

5. [15] Use the substitution $t = 3\tan\theta$ to evaluate the following integral. You will find the identity $\sin(2\theta) = 2\sin\theta\cos\theta$ useful.

$$\int \frac{dt}{(t^2+9)^2}$$

Let $t = 3\tan\theta$

so $dt = 3\sec^2\theta\,d\theta$

& $t^2+9 = 9\tan^2\theta + 9 = 9\sec^2\theta$

$$= \int \frac{3\sec^2\theta\,d\theta}{81\sec^4\theta} = \frac{1}{27}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{54}\int(1+\cos 2\theta)\,d\theta = \frac{1}{54}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C = \frac{1}{54}\left(\theta + \sin\theta\cos\theta\right) + C$$

$$= \frac{1}{54}\left[\arctan\left(\frac{t}{3}\right) + \frac{3t}{t^2+9}\right] + C$$