1. [5] Express the following in its partial fraction form. You do not need to solve for the coefficients.

\[
\frac{1}{(x+1)^3(x^2+1)}
\]

2. Integrate.

(a) [10] \( \int \frac{5x^2 + x + 3}{x(x^2 + 1)} \, dx \)

(b) [10] \( \int \frac{2x + 5}{x^2 + 4x + 5} \, dx \)
3. [10] Evaluate. \[ \int_0^1 \ln x \, dx \]

4. [10] Find a constant \( C \) such that \( p(x) \) is a probability density function on the given interval, and compute the probability indicated.

\[
p(x) = \frac{C}{(x + 1)^3} \quad \text{on} [0, \infty); \quad P(0 \leq X \leq 1)
\]
5. [10] Use the Comparison Test to show \( \int_{1}^{\infty} \frac{dx}{(x+2)(x+3)} \) converges.

6. [10] Show that for \( R > 1, \)

\[
\int_{1}^{R} \frac{dx}{(x+2)(x+3)} = \ln \left| \frac{R+2}{R+3} \right| - \ln \frac{3}{4}
\]

7. [5] Show \( \int_{1}^{\infty} \frac{dx}{(x+2)(x+3)} = \ln \frac{4}{3} \)
8. [10] Find the arc length of \( y = \frac{x^3}{12} + \frac{1}{x} \) for \( 1 \leq x \leq 2 \). \textit{Hint:} Show that \( 1 + (y')^2 \) is a perfect square.

9. [10] Show the surface area of a sphere of radius \( R \) is \( S = 4\pi R^2 \) by computing the surface area generated by rotating the semicircle \( y = \sqrt{R^2 - x^2} \) about the \( x \)-axis.