

# Math 105 Final Exam December 13, 2006

Chapters P2 – 5.1

Name \_\_\_\_\_ Instructor name or section # \_\_\_\_\_

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	100
4	9	15	8	9	12	3	6	10	4	3	6	5	5	5	6	5	4	3	8	8	3	6	3	

Given

QF:  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard form for a circle:  $(x - h)^2 + (y - k)^2 = r^2$

midpoint of a line  $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

distance formula  $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

point-slope form:  $y = m(x - x_1) + y_1$

Odd:  $f(-x) = -f(x)$

average rate of change:  $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

periodic compounding  $A = P \left( 1 + \frac{r}{n} \right)^{nt}$

Even:  $f(x) = f(-x)$

continuous compounding  $A = Pe^{rt}$

Properties of logs:  $\ln(a \cdot b) = \ln a + \ln b$ ,  $\ln \frac{a}{b} = \ln a - \ln b$ ,  $\ln a^p = p \ln a$

**Show All Work for Full Credit!!**

1. (2 pts ea) Completely factor the following and simplify if possible. Do NOT solve for x.

a.)  $4x^3 + 4x^2 + 4x + 4 = 0$

b.)  $4x^3 = 400x$

2. (3 pts ea) Use interval notation to express the solution set for:

a.)  $-4x < 20$

b.)  $9|x + 2| \geq 27$

c.)  $4 < x - 2 < 8$

3. (3 pts ea) Solve the following for x by the method of your choice (beware of extraneous solutions).

a.)  $6x - (4x - 5) = 15$

b.)  $6x^{3/2} - 42 = 0$

c.)  $\sqrt{2x+5} = x-5$

d.)  $-3|2x+2| = -6$

e.)  $\frac{x+6}{2x} + 1 = \frac{x+3}{x}$

4. (2 pts ea) Find the exact value of the logarithm without using a calculator.

a.)  $\log_8 64 =$

b.)  $\log_2 \frac{1}{8} =$

c.)  $\ln 1 =$

d.)  $\ln(e^4) =$

5. (3 pts ea) Use properties of logarithms to expand (a and b) or condense (c) the logarithmic expression as much as possible. Do NOT solve for x, y or z.

a.)  $\log_8 \left( \frac{x^3 y}{\sqrt{z}} \right)$

b.)  $\log_4 \sqrt[3]{\frac{x}{y}}$

c.)  $7\ln x - \frac{1}{5}\ln y$

6. (3 pts ea) Solve the following equations using properties of logs or exponentials. Express the answer to 2 decimals or in terms of logarithms or exponentials.

a.)  $2e^{5x} = 1262$

b.)  $2 + 3\ln x = 20$

c.)  $\log_2 x + \log_2(x+15) = 4$

d.)  $\log_2(x + 3) - \log_2(x - 3) = 2$

7. (3 pts) Skeletons were found at a construction site in San Francisco recently which would have held 50 kg of carbon-14 when the animal died. Now they contained 36.5 kg. Use the exponential decay model for carbon-14,  $A = A_0e^{-0.000121t}$ , to determine how old the skeletons were when found.

8. (2 pts ea) The logistic growth function,  $f(t)$ , describes the number of people,  $f(t)$ , who have become ill with influenza  $t$ -weeks after its initial outbreak in a particular community.  $f(t) = \frac{105,000}{1 + 4100e^{-t}}$ .

How many people were initially infected? \_\_\_\_\_

How many people were ill at the end of the 4<sup>th</sup> week? \_\_\_\_\_

What is the limiting size of the population that becomes ill? \_\_\_\_\_

9. (10 pts) Given  $f(x) = 3x + 4$ , find and simplify the following:

a.)  $f(2) =$

b.)  $f(m) =$

c.  $\frac{f(x+h) - f(x)}{h} =$

d.)  $f^{-1}(x) =$

10. (2 pts ea) Given  $h(x) = (f \circ g)(x)$  where  $h(x) = (x + 3)^3$  show

$f(x) = \underline{\hspace{2cm}}$ ,  $g(x) = \underline{\hspace{2cm}}$

11. (1 pt ea) Given the piecewise function,  $g(x)$ , evaluate at the three points below.

$g(x) = \begin{cases} 5x + 5 & x \geq 0 \\ x - 3 & x < 0 \end{cases}$   $g(-4) = \underline{\hspace{2cm}}$   $g(0) = \underline{\hspace{2cm}}$   $g(1) = \underline{\hspace{2cm}}$

12. (3 pts ea) Write the equations for the lines passing through (4, 2) parallel to  $y = -2x - 4$ .

Point-slope form:  $\underline{\hspace{4cm}}$

Slope-intercept form:  $\underline{\hspace{4cm}}$

13. (5 pts) Given  $f(x) = \frac{x^2 - 144}{x^2 + 144}$ . Find the x-intercepts:  $x = \underline{\hspace{2cm}}$ ,

y-intercepts:  $y = \underline{\hspace{2cm}}$ ,

vertical asymptotes:  $x = \underline{\hspace{2cm}}$ .

horizontal asymptotes:  $y = \underline{\hspace{2cm}}$

14. (3 pts) a.) You are choosing between health clubs. LetitBme offers membership for a fee of \$20 plus a monthly fee of \$14. BeTough offers membership for a fee of \$14 plus a monthly fee of \$17. After how many months will the total cost for the two health clubs be the same and what is the total cost? Show work. Guess and check answers will receive NO points.

b.) (2 pts) Solve for t.  $M = G + Grt$

15. Complete the square (3 pts) and write the equation of the circle in standard form. Indicate the center and the radius.  $x^2 + y^2 - 8x + 7 = 0$

circle eqn: \_\_\_\_\_ (1 pt ea) center (\_\_\_\_, \_\_\_\_ ) radius \_\_\_\_\_

16. (3 pts ea) The diameter of a circle has endpoints (4, 4) and (6, 8). Find the radius and the center.  
center: \_\_\_\_\_ radius: \_\_\_\_\_

17. Find the vertex (3 pts). Circle below whether that vertex is a maximum or a minimum. (2 pts)

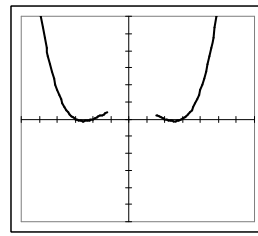
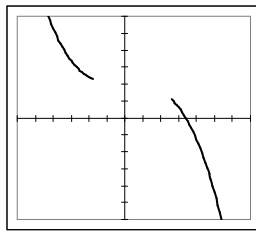
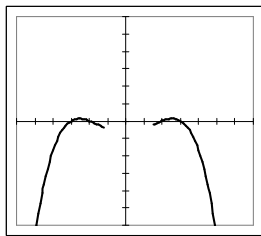
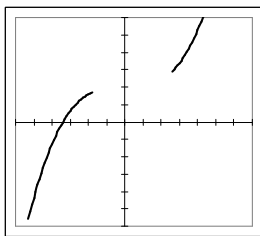
$$f(x) = -2x^2 - 12x + 122$$

Vertex \_\_\_\_\_ min or max?

18. (2 pts ea) Connect the function to its end behavior.

a.  $f(x) = 35 - x^3$

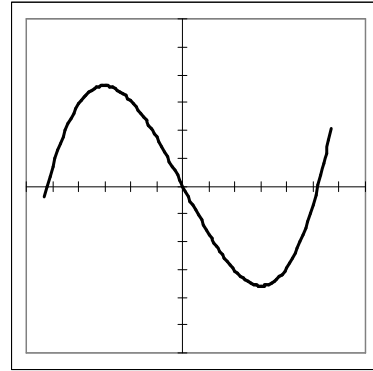
b.  $f(x) = -5(x^2 - 9)(x^2 - 4)$



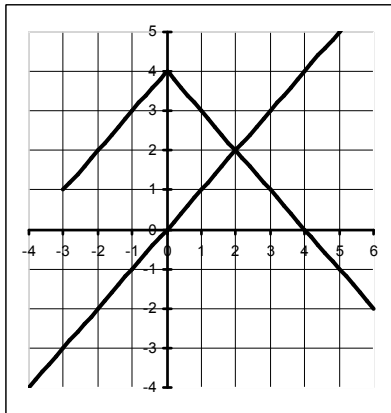
19. (3 pts) Show why the function is even, odd, or neither:  $f(x) = x^5 - x^3 + 1$

20. (2 pts ea) Use the graph of  $f(x)$  to determine the following.

- a.) x-value at which  $f(x)$  has a relative min: \_\_\_\_\_  
 b.) x-value at which  $f(x)$  has a relative max: \_\_\_\_\_  
 c) Regions of increasing: \_\_\_\_\_  
 d) Regions of decreasing: \_\_\_\_\_



21.



**f**

**g**

**g**

(2 pts ea)

b.)  $\left(\frac{g}{f}\right)(3) = \underline{\hspace{2cm}}$

c.)  $(g \circ f)(0) = \underline{\hspace{2cm}}$

**f**

d.)  $(g - f)(-1) = \underline{\hspace{2cm}}$

e.) if  $f(x) = 2$ , then  $x = \underline{\hspace{2cm}}$

22. (3 pts) Use division and the remainder theorem to find  $f(-4)$  for  $4x^3 - 5x^2 - 3x + 3 = f(x)$

$f(-4) = \underline{\hspace{2cm}}$

23. (2 pts ea) Show the domain in interval or set builder notation of

a.)  $f(x) = \frac{6}{x+4} + \frac{5}{x-3}$ ,

b.)  $f(x) = \sqrt{x-2}$

c.)  $f(x) = \ln(x+1)$

24. (3 pts) Solve this system of equations for  $x$  and  $y$  using substitution or addition. The method is important. Guess and check will receive no credit.  $2x - y = 3$ ,  $3x + 2y = 8$