Exam 2 100 points

Statistics 401

November 10, 2016

Name KEY

TRUE/FALSE: (2 pts each) For each of the following, circle T or F.

1. **T / F** If X is the length of a rod shaped *Pseudomonas aeruginosa* bacterium, and if $X \sim N(\mu = 6.5\mu m, \sigma = 0.1\mu m)$, then 68% of these bacteria are between 6.4 and 6.6 μm long.
2. **T / F** A farmer owns a population of $N = 50$ cows. He randomly chooses $n = 5$ cows without replacement from the herd and carefully monitors X, gestation length, for each of these five. If $\mu_x = 281$ days and $\sigma_x = 13$ days, then $\sigma_{\bar{x}} = \frac{13}{\sqrt{5}}$.
3. **T / F** The population standard deviation σ decreases as the sample size n increases.
4. **T / F** When choosing between an unbiased and a biased statistic, always choose the unbiased one.
5. **T / F** If a 95% confidence interval for μ, the mean fuel efficiency for a new SUV, is (17.21mpg, 19.75mpg), then μ is larger than 19.75 mpg about 2.5% of the time.
6. **T / F** As the confidence level increases, the width of the confidence interval increases.
7. **T / F** As the sample size n increases, the t distribution $t(n-1)$ converges to a normal distribution.
8. **T / F** The following is a valid pair of statistical hypotheses: $\text{H}_0\colon \mu > 0.6$ versus $\text{H}_a\colon \mu = 0.6$.
9. **T / F** The following is a valid pair of statistical hypotheses: $\text{H}_0\colon \overline{X} = 4$ versus $\text{H}_a\colon \overline{X} > 4$.
10. **T / F** A p-value is a probability.
11. **T / F** Rejecting H_0 implies H_a is true.
12. **T / F** The larger the p-value, the stronger the evidence that H_0 is true.
13. **T / F** A t distribution has thicker tails than the standard normal distribution.

14. (8 pts) State the Central Limit Theorem. Be sure to state the assumptions and conclusion.

If n is large, $X_1, \ldots, X_n$ is a SRS, an if N is large, then

$$\overline{X} \sim N(\mu, \sigma/\sqrt{n}).$$

MULTIPLE CHOICE: (4 pts each) Circle the single best answer.

15. When concerned about normality of the sampling distribution of $\overline{X}$, one should consider transforming a sample if

  A. The sample size is large and the data comes from an approximately normal distribution.
  B. The sample size is large and the data does not come from a normal distribution.
  C. The population variance is unknown and you must estimate it.
  D. The sample size is small and the data comes from an approximately normal distribution.
  E. The sample size is small and the data does not come from a normal distribution.

16. For a random sample of $n = 9$ women, the average resting pulse rate is $\bar{x} = 76$ beats per minute and the sample standard deviation is $s = 5$ beats per minute. The *standard error* of the sample mean is

  A. 0.556
  B. 0.745
  C. 1.667
  D. 3.333

$5/\sqrt{n} = 5/\sqrt{9} = 5/3$

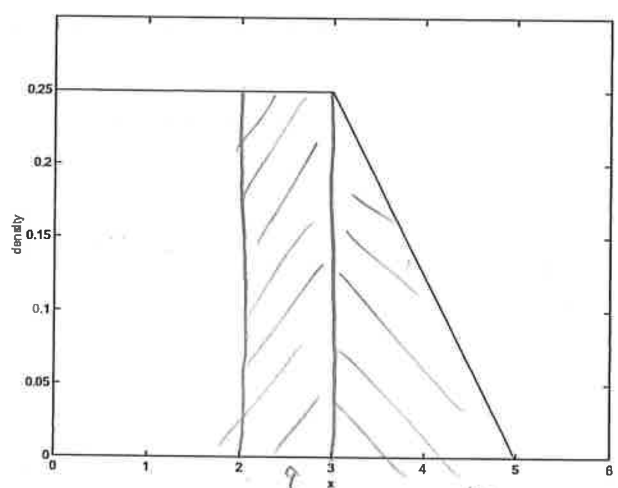
17. Before modern day medicine, lengths of human pregnancies followed a normal distribution with $\mu = 9$ months and $\sigma = 0.5$ months. How long were the longest 5% of pregnancies?

  A. 9.5 months
  B. 9.82 months
  C. 9.98 months
  D. 10.5 months

95th percentile $= \mu + 1.645\sigma$
$\stackrel{?}{=} 9 + 1.645(1/2)$

18. The plot below displays the density of a continuous variable X. The probability $P(X > 2)$ is

  (a) 0
  (b) $\frac{1}{4}$
  (c) $\frac{1}{2}$
  (d) $\frac{3}{4}$



Area = h×b
= 1/4×1
= 1/4

Area = 1/2 h×b
= 1/2×1/4×2
= 1/4

SHORT ANSWER AND COMPUTATION: Show all work to receive full credit! Write legibly.

19. (10 pts) Five temperature measurements, X, (in Fahrenheit) are taken from an underground lake in Antarctica. In order to satisfy the assumptions to construct a valid CI, the investigator decides to take the reciprocal transform of the data:

$$Y = X^{-1}.$$

She then computes a 95% CI for μ_Y and gets (.026, .032).

(a) Give a 95% CI for the population median $\tilde{\mu}_X$. SHOW YOUR WORK!

Backtransforming the CI for μ_Y:

$$(0.026, 0.032)^{1/-1} = \left(1/.032 , 1/.026 \right)$$

$$\approx (31.25, 38.46)$$

(b) Interpret the CI in (19a) **in terms of the problem.**

We are 95% confident that the true median temperature is between 31.3°F and 38.5°F.

20. (8 pts) A real estate agent would like to estimate the proportion of offices that are currently occupied in a large city. How large of a sample is required to ensure that the estimate will have a margin of error of 0.1 with 98% confidence? SHOW YOUR WORK!

$$n = p(1-p)\left[\frac{z_{.99}}{m}\right]^2 = (1/2)(1/2)\left[\frac{2.326}{0.1}\right]^2$$

$$= 135.3$$

Therefore, n = 136 offices should be included in the sample.

21. An industrial plant claims to discharge no more than 1000 gallons of waste water per hour, on the average, into a neighboring lake. The Environmental Protection Agency decides to monitor the plant, in case this limit is being exceeded. Based on a random sample of size $n = 44$ they find $\bar{x} = 1021$ gallons and $s = 200$ gallons.

(a) (2 pts) What type of study is this? Circle one of the following:

Observational Study	Experiment

Because a treatment is not being imposed.

(b) (6 pts) State the null and alternative hypotheses.

$H_0: \mu = 1000$

$H_a: \mu > 1000$

(c) (6 pts) What assumptions do you need to check before conducting a hypothesis test? Are these assumptions satisfied? SHOW YOUR WORK!

SRS – problem statement confirms this.

n large – $n = 44 \geq 30$, so n is large

N large – N is potentially infinite

(d) Conduct the hypothesis test.

i. (4 pts) Calculate the value of the test statistic. SHOW YOUR WORK!

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1021 - 1000}{200/\sqrt{44}} = 0.696$$

ii. (4 pts) Give the p-value. Indicate whether you are using the z-table or the t-table. If the t-table, how many degrees of freedom are you using?

p-value $= p(T > .696) > 0.20$, found via t distribution with $df = 40$ (not $df = n-1 = 43$)

iii. (3 pts) Make a decision. Use a significance level of $\alpha = .05$. Justify your answer.

Because p-value $> 0.20 > .05 = \alpha$, FTR H_0!

iv. (5 pts) Make a conclusion **in terms of the problem.**

The evidence fails to suggest that the mean waste water discharge is larger than 1000 g/h.