```
1-Sample Z Test

Testing mean = null (versus > null)
Alpha = 0.01     Sigma = 0.2     Sample Size = 36

Difference     Power
      0.10    0.7497
      0.15    0.9851
```

The slight differences between the power values computed by MINITAB and those previously obtained are due to rounding in Example 10.17.

The probability of a Type II error and the power for z tests concerning a population proportion are calculated in an analogous manner.

▪ Example 10.18 Power for Testing Hypotheses About Proportions

A package delivery service advertises that at least 90% of all packages brought to its office by 9 A.M. for delivery in the same city are delivered by noon that day. Let π denote the proportion of all such packages actually delivered by noon. The hypotheses of interest are

$$H_0\colon\ \pi = .9 \qquad \text{versus} \qquad H_a\colon\ \pi < .9$$

where the alternative hypothesis states that the company's claim is untrue. The value $\pi = .8$ represents a substantial departure from the company's claim. If the hypotheses are tested at level .01 using a sample of $n = 225$ packages, what is the probability that the departure from H_0 represented by this alternative value will go undetected?

At significance level .01, H_0 is rejected if P-value $\leq .01$. For the case of a lower-tailed test, this is the same as rejecting H_0 if

$$z = \frac{p - \mu_p}{\sigma_p} = \frac{p - .9}{\sqrt{\frac{(.9)(.1)}{225}}} = \frac{p - .9}{.02} \leq -2.33$$

(Because -2.33 captures a lower-tail z curve area of .01, the smallest 1% of all z values satisfy $z \leq -2.33$.) This inequality is equivalent to $p \leq .853$, so H_0 is *not* rejected if $p > .853$. When $\pi = .8$, p has approximately a normal distribution with

$$\mu_p = 0.8 \qquad \sigma_p = \sqrt{\frac{(.8)(.2)}{225}} = 0.0267$$

Then β is the probability of obtaining a sample proportion greater than .853, as illustrated in Figure 10.6.

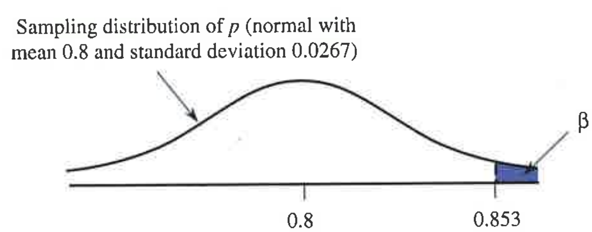

Figure 10.6 β when $\pi = .8$ in Example 10.18.

Converting to a z score results in

$$z = \frac{0.853 - 0.8}{0.0267} = 1.99$$

and Appendix Table 2 gives

$$\beta = 1 - .9767 = .0233$$

When $\pi = .8$ and a level .01 test is used, less than 3% of all samples of size $n = 225$ result in a Type II error. The power of the test at $\pi = .8$ is $1 - .0233 = .9767$. This means that the probability of rejecting H_0: $\pi = .9$ in favor of H_a: $\pi < .9$ when π is really .8 is .9767, which is quite high.

▪ β and Power for the t Test (Optional)

The power and β values for t tests can be determined by using a set of curves specially constructed for this purpose or by utilizing appropriate software. As with the z test, the value of β depends not only on the true value of μ but also on the selected significance level α; β increases as α is made smaller. In addition, β depends on the number of degrees of freedom, $n - 1$. For any fixed level α, it should be easier for the test to detect a specific departure from H_0 when n is large than when n is small. This is indeed the case; for a fixed alternative value, β decreases as $n - 1$ increases.

Unfortunately, there is one other quantity on which β depends: the population standard deviation σ. As σ increases, so does $\sigma_{\bar{x}}$. This in turn makes it more likely that an $\bar{x}$ value far from μ will be observed, resulting in an erroneous conclusion. Once α is specified and n is fixed, the determination of β at a particular alternative value of μ requires that a value of σ be chosen, because each different value of σ yields a different value of β. (This did not present a problem with the z test because when using a z test, the value of σ is known.) If the investigator can specify a range of plausible values for σ, then using the largest such value will give a pessimistic β (one on the high side) and a pessimistic value of power (one on the low side).

Figure 10.7 shows three different β curves for a one-tailed t test (appropriate for H_a: $\mu >$ hypothesized value or for H_a: $\mu <$ hypothesized value). A more complete

Figure 10.7 β curves for the one-tailed t test.