Homework #6

Due: Friday, March 24, 2017

John Muir (1838-1914):

Pursuing my lonely way down the valley, I turned again and again to gaze on the glorious picture, throwing up my arms to inclose it as in a frame. After long ages of growth in the darkness beneath the glaciers, through sunshine and storms, it seemed now to be ready and waiting for the elected artist, like yellow wheat for the reaper; and I could not help wishing that I were that artist. I had to be content, however, to take it into my soul.

1. List the 7 beautiful properties of MLEs.

2. Do Exercise 9.98. This problem asks you to (again) consider a random sample $y_1, ..., y_n \sim \text{Geometric}(p)$ distribution as in Exercise 9.97. Let $\hat{p}$ be the MLE of $p$ from 9.97b. This problem is asking you to find the asymptotic variance (i.e., using Cramer-Rao) of $\hat{p}$.

3. Find a large sample 95% CI for a Geometric parameter $p$ using $\hat{p}$ and the variance from problem #1.

4. Do Exercise 9.99. This problem is asking you to find a 95% CI for a binomial proportion $p (=t(p))$ using the asymptotic (Cramer-Rao) variance.

5. Do Exercise 9.100. Break up the problem into the following parts:
   (a) State the MLE of $\theta^2$ based on your previous work for #4 in HW5.
   (b) Find the asymptotic variance of the MLE of $\theta^2$.
   (c) Find a large sample 95% CI for $\theta^2$.

6. Do Exercise 9.101. Maybe this notation is useful: let $\theta = t(\lambda) = e^{-\lambda}$ where $\lambda$ is the Poisson parameter, and let $\hat{\theta} = t(\hat{\lambda})$ be the MLE for $\theta$. Break up the problem into the following parts:
   (a) State the MLE for $e^{-\lambda}$ based on your previous work for Exercise 9.80d.
   (b) Find the asymptotic variance of the MLE ($\hat{\theta}$) of $e^{-\lambda}$.
   (c) Find a large sample 95% CI for $e^{-\lambda}$. 