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AI Parker

Data with a Map Seeking Circuit

Tracking Eye Motion from Retinal Scan
Joint work with

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Math Dept
The Algorithm which finds the motion: Map-Seeking Circuit

- Image Registration
- Estimating Eye Motion from AO SL data

Show and Tell (Results)
Results: Estimated Motion from AOSSO Data
1. Smooth the data with a Gaussian kernel to reduce the effect of noise and amplitude variation.
2. Break the smoothed image into "channels".
3. Determine the eye motion across patches.

This yields HIGH RESOLUTION of the motion, which is about 480 - 960 estimates per second.

This yields HIGH RESOLUTION of the motion, which is about 480 - 960 estimates per frame. We can calculate...
Add each dewarped frame to create a mosaic or montage.

Dewarp each frame of the AOSLO video.
How to add frames: Initialize montage with (intensity=0, weight=0)
How to add frames: Map the first pixel, update the montage.
How to add frames: Map the next pixel, update the montage...
image $\mathcal{F}$ and let $L^i$ be some vertical translation of the image.

For example, we can let $L^i$ be some horizontal translation of the image $\mathcal{F}$ and let $\mathbf{F}$ be some vector in Hilbert spaces $\mathcal{H}$ and $\mathcal{H}^0$, respectively. Given transformation $T$:

$$T : \mathcal{H} \rightarrow \mathcal{H}^0$$

where for each "layer" $\ell$ between 1 and $L$, we have

$$\{ \forall \ell \in \mathcal{L} \}$$

where for each "layer" $\ell$ between 1 and $L$, we have

$$L^1 \circ L^2 \circ \cdots \circ L^L = L$$

formations of form

Goal: Find $T$ which maximizes correspondence from linear trans-

$$\mathcal{H} \langle \mathcal{F}, (\mathcal{F}) T \rangle$$

inner product to be the inner product

denote between $\mathcal{F}$ and $\mathcal{F}^T$ as vectors in Hilbert spaces $\mathcal{H}$ and $\mathcal{H}^0$, respectively. Given transformation $T$:

$$\mathcal{H} : \mathcal{H} \rightarrow \mathcal{H}^0$$

Model images $\mathcal{F}$, $\mathcal{F}^T$ as vectors in Hilbert spaces $\mathcal{H}$ and $\mathcal{H}^0$, respectively.
ADVANTAGE of MSC over Cross-Correlation: MSC can include other transformations such as rotations, dilations, shear, compression, and so forth.

SYNOPSIS: A Map-Seeking Circuit finds a solution to the discrete optimization problem:

\[
\mathcal{H} \left( \mathcal{F}, (\mathcal{G}) (I_1, I_2, \ldots, I_L) \right) \max_{\mathbf{I}} \arg \max \mathcal{H} = (I_{*1}, \ldots, I_{*L})
\]
Imbed the discrete problem in continuous constrained optimization problem. Maximize multilinear form. Where

\[ \langle \mathcal{E} (T)x \rangle_{\mathcal{H}} \circ \cdots \circ (T+\eta)x \mathcal{L} \circ \cdots \circ (T-\eta)x \mathcal{L} \circ \eta \mathcal{L} \rangle = \frac{\langle \eta x \mathcal{E} \rangle}{\mathcal{W} \mathcal{E}} \]

Simplifying Property: Components of gradient of \( M \) can be computed quickly and relatively cheaply via the inner product.

\[ \langle \eta^i \rangle \mathcal{L} \langle \eta^i \rangle x \sum_{j=1}^{I} = \langle \eta^i \rangle x \mathcal{L} \]

MSC KEY IDEA (which makes it fast)
where $T^*$ is the adjoint or conjugate transpose of $T$.

$T^0$ is the adjoint or conjugate transpose.

MSC can be viewed as an iterative algorithm which uses this gradient to maximize the correspondence.
The computational complexity of each MSC-type iteration is of the form

\[ \sum_{i=1}^{L} n_i \times \cdot \cdot \cdot \times n_i \]

where \( n_i \) is the number of transformations in layer \( i \), and \( L \) is the number of layers. The complexity of an exhaustive search is the product of the order of the sum of the computational complexity of each MSC-type iteration.
CONCLUSIONS

Using MSC,

We can accurately track eye motion from AO-SLO videos,

Register AO-SLO video frames to create a de-noised retinal image

video frames, including shear and compression, even through saccades.

We can identify very general transformations between AO-SLO videos,

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