

① Circle True or False for the following statements.

a) If  $\sum_{n=1}^{\infty} a_n$  converges then the sequence  $\{a_n\}$  converges.

TRUE

FALSE

b) If  $\lim_{n \rightarrow \infty} a_n = 2$  then  $\sum_{n=1}^{\infty} a_n$  diverges.

TRUE

FALSE

c) The series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  diverges.

TRUE

FALSE

d) If  $\sum_{n=1}^{\infty} (a_n + b_n)$  converges then both  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  must converge.

TRUE

FALSE

e) If  $a_n > b_n$  for all  $n$  and  $\sum a_n$  converges then  $\sum b_n$  must converge by the comparison test.

TRUE

FALSE

Circle True or False for the following statements.

a) If  $\sum_{n=1}^{\infty} a_n$  converges then the sequence  $\{a_n\}$  converges.

TRUE

FALSE

$a_n \rightarrow 0$  necessarily

b) If  $\lim_{n \rightarrow \infty} a_n = 2$  then  $\sum_{n=1}^{\infty} a_n$  diverges.

TRUE

FALSE

$a_n \not\rightarrow 0$  hence series diverges

c) The series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  diverges.

TRUE

FALSE

p-series,  $p = \frac{1}{2} \leq 1$

d) If  $\sum_{n=1}^{\infty} (a_n + b_n)$  converges then both  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  must converge.

TRUE

FALSE

$a_n = +1$        $b_n = -1$

e) If  $a_n > b_n$  for all  $n$  and  $\sum a_n$  converges then  $\sum b_n$  must converge by the comparison test.

TRUE

FALSE

$\sum b_n$  could be any negative divergent series with  $b_n < a_n$   
Need  $b_n \geq 0$  for comparison test.

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Circle True or False for the following statements.

a) If  $\lim_{n \rightarrow \infty} a_n = 0$  then  $\sum_{n=1}^{\infty} a_n$  converges.

TRUE

FALSE

b) If a sequence  $\{a_n\}$  converges then the series  $\sum_{n=1}^{\infty} a_n$  must converge.

TRUE

FALSE

c) If a series converges conditionally then it does not converge absolutely.

TRUE

FALSE

d) The series  $\sum_{n=1}^{\infty} \frac{n}{n+1}$  diverges.

TRUE

FALSE

e) Suppose that  $b_n > 0$ ,  $\lim_{n \rightarrow \infty} b_n = 0$  but that  $\{b_n\}$  is not a decreasing sequence. Then, the Alternating Series Test implies  $\sum_{n=1}^{\infty} (-1)^n b_n$  diverges.

TRUE

FALSE

Circle True or False for the following statements.

a) If  $\lim_{n \rightarrow \infty} a_n = 0$  then  $\sum_{n=1}^{\infty} a_n$  converges.

TRUE

FALSE

$$\sum_{n \geq 1} \frac{1}{n} \text{ diverges!}$$

b) If a sequence  $\{a_n\}$  converges then the series  $\sum_{n=1}^{\infty} a_n$  must converge.

TRUE

FALSE

$$a_n = \frac{n}{n+1} \rightarrow 1 \neq 0$$

$$\sum_{n \geq 1} \frac{n}{n+1} \text{ diverges!}$$

c) If a series converges conditionally then it does not converge absolutely.

TRUE

FALSE

Definition of conditional convergence

d) The series  $\sum_{n=1}^{\infty} \frac{n}{n+1}$  diverges.

TRUE

FALSE

$$a_n = \frac{n}{n+1} \not\rightarrow 0$$

e) Suppose that  $b_n > 0$ ,  $\lim_{n \rightarrow \infty} b_n = 0$  but that  $\{b_n\}$  is not a decreasing sequence. Then, the Alternating Series Test implies  $\sum_{n=1}^{\infty} (-1)^n b_n$  diverges.

needs  $b_n \downarrow$

Hard. Let

$$b_n = \begin{cases} \frac{1}{n} & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

then

$$S = \frac{1}{2} + \frac{1}{4} + \dots = \frac{1}{2} \sum_{n \geq 1} \frac{1}{n}$$

diverges

TRUE

FALSE