Math 172 , Spring 2013 Section: ________ Name: ______________________

Midterm 1 : Tuesday, February 5, 2013.

Problem	1	2	3	4	5	6	7	8	9	Total
Value	15	10	10	10	10	15	10	10	10	100
Points										

Instructions : All work must be shown to receive full credit.

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1. [15pts] Use the method of substitution to evaluate the following definite integral:

$$\int_0^4 \frac{x\,dx}{\sqrt{x^2+9}}$$

$u = x^2 + 9$

$du = 2x\,dx$

$x = 0 \Rightarrow u = 9$

$x = 4 \Rightarrow u = 25$

$$I = \frac{1}{2}\int_9^{25} u^{-1/2}\,du$$

$$I = u^{1/2}\Big|_9^{25} = \sqrt{25} - \sqrt{9} = 2$$

2. [10pts] Evaluate the following integral

I. B. Parts

$$\int x^2 \ln x\,dx$$

$u = \ln x \qquad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx \qquad dv = x^2 dx$

$$I = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$$

$$I = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

3. [10pts] Evaluate the following indefinite integral

$$\int \frac{dx}{\sqrt{1-16x^2}}$$

$u = 4x$

$I = \frac{1}{4} \int \frac{du}{\sqrt{1-u^2}}$

$I = \frac{1}{4} \arcsin u + C$

$I = \frac{1}{4} \arcsin(4x) + C$

4. [10pts] Evaluate the following integral

$$\int \tan^5 x \sec^4 x \, dx$$

$I = \int \tan^5 x \, (1 + \tan^2 x) \, \underline{\sec^2 x \, dx}$

$u = \tan x$

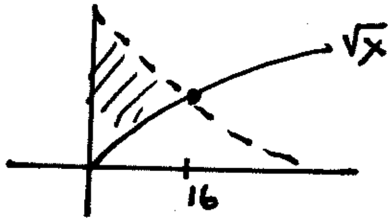
$du = \underline{\sec^2 x \, dx}$

$I = \int (u^5 + u^7) \, du$

$I = \frac{1}{6} u^6 + \frac{1}{8} u^8 + C$

$I = \frac{1}{6} \tan^6 x + \frac{1}{8} \tan^8 x + C$

5. [10pts] Find the area of the region bounded by the curves $y = 8 - \sqrt{x}$, $y = \sqrt{x}$ and $x = 0$



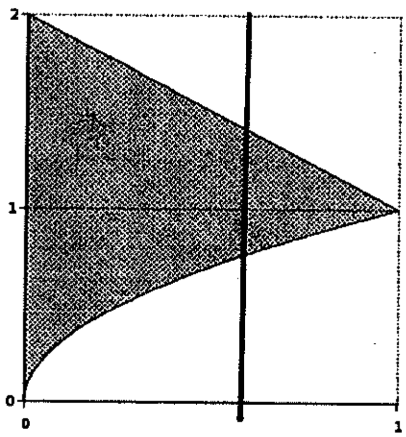
$$8 - \sqrt{x} = \sqrt{x}$$

$$x = 16$$

$$A = \int_0^{16} (8 - 2\sqrt{x})\,dx = 8x - \tfrac{4}{3}x^{3/2} \Big|_0^{16}$$

$$A = 8(16) - \tfrac{4}{3}\cdot 4^3 = 128 - \frac{256}{3} = \frac{128}{3}$$

6. [15pts] A region R bounded by $y = 2 - x$ and $y = \sqrt{x}$ with $x > 0$ is revolved about the x-axis. Use the disk/washer method to compute the volume of the resulting solid. See Figure below:



$$V = \int_0^1 \pi(2-x)^2 - \pi(\sqrt{x})^2\,dx$$

$$V = \pi \int_0^1 (x^2 - 5x + 4)\,dx$$

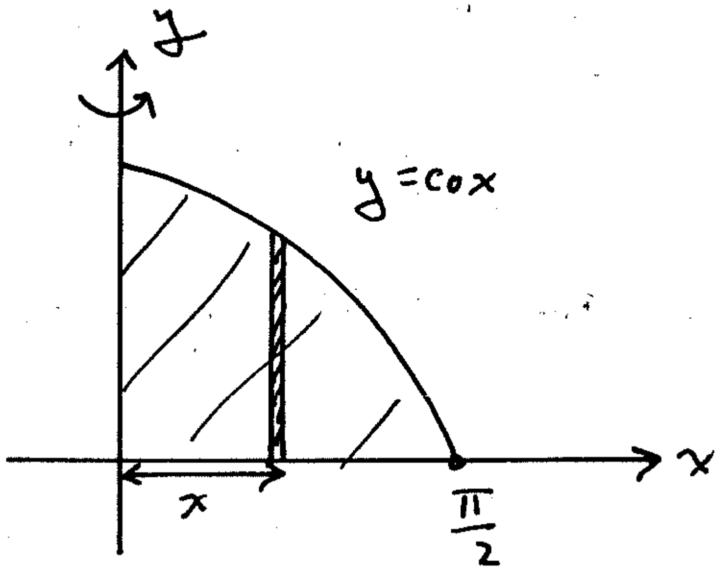
$$V = \pi\left(\tfrac{1}{3}x^3 - \tfrac{5}{2}x^2 + 4x\right)\Big|_0^1$$

$$V = \pi\left(\tfrac{1}{3} - \tfrac{5}{2} + 4\right)$$

$$V = \frac{11\pi}{6}$$

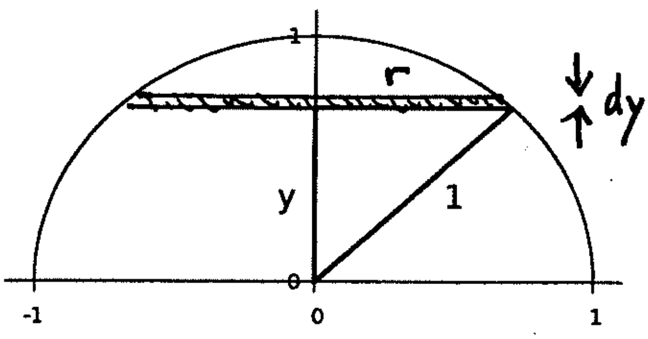
7. [10pts] A region R is bounded by $y = \cos x, y = 0$ and $x = 0$ and has $0 < x < \frac{\pi}{2}$. A solid is formed by revolving the region about the y-axis.

Draw the region R and then write out a definite integral for the solid's volume using the method of cylindrical shells. DO NOT evaluate the integral.



$$V = 2\pi \int_0^{\pi/2} x \cos x \, dx$$

8. [10pts] Illustrated below is the cross section of a hemisphere of radius 1 meter. The hemisphere is full of water of density $\rho = 1000kg/m^3$. Compute the work done by pumping all of the water out the **top**. You may leave your answer as a multiple of ρ and the gravitational constant $g = 9.8m/sec^2$.



$$r = \sqrt{1-y^2}$$

Mass of slice

$$dm = \pi r^2 \rho \, dy$$

$$\underbrace{dm = \pi\rho(1-y^2)dy}$$

when pumped a slice of mass dm moves a distance $(1-y)$ to the top. Work done for all slices

$$W = g\int_0^1 \pi\rho(1-y^2)(1-y)\,dy$$

$$W = \rho\pi g \int_0^1 (1-y^2)(1-y)\,dy$$

Some messy calculations

$$W = \rho\pi g \cdot \frac{5}{12}$$

9. [10pts] Compute the average of $f(x) = \sin^2(2x)\cos(x)$ over $[0, \pi]$.

Are many ways.

$$A = \frac{1}{\pi}\int_0^{\pi} \sin^2(2x)\cos x \, dx$$

$$A = \frac{1}{\pi}\int_0^{\pi} (2\sin x \cos x)^2 \cos x \, dx$$

$$A = \frac{1}{\pi}\int_0^{\pi} 4\sin^2 x \underbrace{(1-\sin^2 x)}_{\cos^2 x} \cos x \, dx$$

Let $u = \sin x$, $du = \cos x \, dx$. Indef integral

$$A_i = \frac{4}{\pi}\int (u^2 - u^4)\, du$$

$$A_i = \frac{4}{\pi}\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right) + C$$

To conclude

$$A = \frac{4}{\pi}\left[\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x\right]\Big|_0^{\pi}$$

$$A = 0$$

Indefinite integrals and Trigonometric Identities

Some trigonometric identities which may or may not be needed include:

$$\begin{aligned}
\cos^2 x &= \frac{1}{2}(1+\cos 2x) \\
\sin^2 x &= \frac{1}{2}(1-\cos 2x) \\
\sin 2x &= 2\ \sin x\ \cos x
\end{aligned}$$

$$\begin{aligned}
\sin A\ \cos B &= \frac{1}{2}(\sin(A+B)+\sin(A-B)) \\
\cos A\ \cos B &= \frac{1}{2}(\cos(A+B)+\cos(A-B)) \\
\sin A\ \sin B &= \frac{1}{2}(\cos(A-B)-\cos(A+B))
\end{aligned}$$

Some integrals which may or may not be needed include:

$$\begin{aligned}
\int \frac{du}{1+u^2} &= \arctan u + c \\
\int \frac{du}{\sqrt{1-u^2}} &= \arcsin u + c \\
\int \sec u\ du &= \ln|\sec u + \tan u| + c \\
\int \tan u\ du &= \ln|\sec u| + c
\end{aligned}$$